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Special Technical Report for the period October 1978 to May 1982

User's Manual for the Flow-Field Diagnostics Code EMABIC

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prepared for the: **Air Force Rocket Propulsion Laboratory**

Air Force Space Technology Center
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FOREWORD

This manual was submitted by the Aerospace Corporation, El Segundo, California 90245, under Contract No. F04701-81-C-0082 with the Air Force Rocket Propulsion Laboratory, Edwards AFB, California 93523, under Air Force Project Task 573010CU. The manual gives detailed instructions for running the Emission/Absorption Inversion Code (EMABIC).

This report has been reviewed and is approved for publication in accordance with the distribution statement on the cover and on the DD Form 1473.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

EMABIC is a system of codes designed for infrared radiation prediction and flow-field retrieval diagnostics of low-visibility propellant, tactical rocket motors plumes. Radiation prediction employs a coupled gas band model, single-scattering particle model. Retrieval is made for both gas and particle radial properties by iterative Abel inversion of transverse emission, transmittance, and laser scattering profiles obtained at two spectral positions where, respectively, both gas and particles radiate and absorb and where only

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particles radiate and absorb. EMABIC consists of six main programs, 45 sub-programs, and three implied date transfer files. Brief descriptions of all of the codes are given, and detailed data preparation instructions for the six main programs are included. An example application to a minimum smoke propellant plume model containing H_2O and Al_2O_3 as the gas and particle phases is made.

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VARIABLES

Coordinates

r	plume, radial coordinate
z	plume transverse coordinate
s	primary line-of-sight coordinate
σ	scattering line-of-sight coordinate
θ	scattering angle
ϕ	azimuth around primary line of sight

Radial Gas Variables

$p_g(r)$	total pressure
$c_g(r)$	mole fraction concentration
$T_g(r)$	temperature
$\bar{k}\ (r)$	band model absorption parameter
$1/\delta(r)$	band model line density parameter
$\bar{\gamma}_L(r)$	pressure broadened (Lorentz) line halfwidth
$\bar{\gamma}_D(r)$	Doppler line halfwidth
$B_g(r)$	Planck function at T_g

Radial Particle Variables

$\sigma_a(r)$	absorption cross section
$\sigma_s(r)$	scattering cross section
$d\sigma_s(r,\theta)/d\Omega$	differential scattering cross section
$\alpha(r)$	volume absorption cross section
$\beta(r)$	volume scattering cross section
$\gamma(r)$	volume extinction cross section

$p(r,\theta)$	scattering phase function
$T_p(r)$	temperature
$B_p(r)$	Planck function at T_P

Transverse Emission/Absorption/Scattering Variables

$\bar{N}(z)$	total gas-plus-particle emission
$\bar{\tau}(z)$	total gas-plus-particle transmission
$N_p(z)$	particle-only emission
$\tau_p(z)$	particle-only transmission
$f(\theta,z)$	laser scattering efficiency function

Calculation Variables

$J(r)$	emission source function
$\bar{\kappa}(r)$	absorption source function
$G(z,r)$	emission or absorption kernel function
$\bar{\tau}^{\pm}(z,r)$	gas transmittance at (s,r) intersections
$y^{\pm}(z,r)$	derivative function at (s,r) intersections
$\tau_{\alpha}^{\pm}(z,r)$	transmittance due to particle absorption at (s,r) intersections
$\tau_{\beta}^{\pm}(z,r)$	transmittance due to particle scattering at (s,r) intersections
$Q_S^{\pm}(z,r)$	scattering source function at (s,r) intersections
$J(r,\theta)$	laser scattering source function
$G(z,r,\theta)$	laser scattering kernel function

Miscellaneous

λ	wavelength
ν	wavenumber

R	plume radius
a	particle radius
$F(r,a)$	particle size distribution
m	complex index of refraction
n	real part of index of refraction
κ	imaginary part of index of refraction

1. INTRODUCTION

EMABIC* is a system of computer codes designed for infrared radiation prediction and inversion diagnostics of low-visibility propellant tactical rocket motor plumes. The prediction capabilities of the system include profiles of plume infrared emission and transmittance transverse to the plume axis from given radial profiles of the plume gas and particle properties (e.g., concentration and temperature). The angular distribution of laser radiation scattered by the particle component of the plume is also predicted. The diagnostic capability is the retrieval of radial gas and particle distributions from observed transverse and laser scattering profiles.

The radiation prediction capabilities of the system derive from the previously developed prediction code EAPROF (Refs. 1 and 2), and the retrieval diagnostics procedure from the gas-only plume inversion code EMABIC (Ref. 3). The present system supersedes both of these codes.** The diagnostic

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* EMABIC is an acronym for Emission/Absorption Inversion Codes.

1. S. J. Young, Retrieval of Flow-Field Gas Temperature and Concentration of Low-Visibility Propellant Rocket Exhaust Plumes, AFRPL-TR-82-13, U. S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, California, March 1982.

2. S. J. Young, User's Manual for the Plume Signature Code EAPROF, AFRPL-TR-81-08, U. S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, California, January 1981.

3. S. J. Young, Inversion of Plume Radiance and Absorptance Data for Temperature and Concentration, AFRPL-TR-78-60, U. S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, California, 29 September 1978.

** Except, the gas-only diagnostics code with error propagation capability, as described in Ref. 4, is still maintained.

4. S. J. Young, Random Error Propagation Analysis in the Plume Diagnostic Code EMABIC, AFRPL-TR-81-59, U. S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, California, July 1981.

procedures for retrieving particle properties and an improved procedure for gas property retrieval with account of particles are developed in Ref. 5. Reference 5 is also the primary companion report for this user's manual and should be consulted for details not mentioned here.

The primary motivation for EMABIC was the development of the diagnostics tools for retrieving the radial particle and gas properties. The prediction capabilities of EMABIC follow as procedures required in the iterative method of retrieval, and as a means of generating synthetic data with which to test the diagnostic tools. Application of the diagnostic scheme requires traditional transverse infrared emission/absorption (E/A) data at two wavelengths and angular scattering data at one of the wavelengths. λ_1 is a spectral position at which both the gas and particle species of interest radiate and absorb. λ_2 is a position at which only the particles radiate and absorb. The transverse E/A data at these wavelengths are

$$\left.\begin{array}{l}\bar{N}(z)\\ \bar{\tau}(z)\end{array}\right\} \lambda = \lambda_1$$

$$\left.\begin{array}{l}N_p(z)\\ \tau_p(z)\end{array}\right\} \lambda = \lambda_2$$

where $z(o < z < R)$ is the transverse coordinate, R is the plume radius, N is the plume radiance, and τ is the transmittance of the plume to external radiation. Scattering data are required only at λ_2 and consist of the scattering intensity efficiency function

5. S. J. Young, Retrieval of Flow-Field Temperature and Concentration of Low-Visibility Propellant Rocket Exhaust Plumes, AFRPL-TR-82-38, U. S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, California, February, 1983.

$$f(z,\theta) \quad \left\{ \begin{array}{l} o < \theta < \pi \\ o < z < R \\ \lambda = \lambda_2 \end{array} \right.$$

This function is discussed in Ref. 5.

From these data profiles, the retrieval codes of EMABIC recover the radial plume profiles of

$\alpha(r)$	particle volume absorption cross section
$\beta(r)$	particle volume scattering cross section
$\gamma(r)$	particle volume extinction cross section
$p(r,\theta)$	particle scattering phase function
$T_p(r)$	particle temperature
$T_g(r)$	gas temperature
$c_g(r)$	gas concentration

The prediction codes of EMABIC essentially reverse this process and predict the transverse and angular scattering data from the radial data.

The overall structure of EMABIC is described in Section 2. Presently, the system consists of six basic driver programs and 45 subprogram units. The system is structured for easy expansion to handle envisioned added capability, some of which are mentioned in Section 2. In Section 3, a detailed description of data preparation for the six basic codes is given. An example application for a minimum smoke propellant plume model containing H_2O and Al_2O_3, respectively, as the gas and particle phases, is given in Section 4. This example is taken from Ref. 5. Most of the computational methods employed in the system have been discussed in the references to its predecessor codes (Refs. 1, 2, and 3). One significant change is the Abel inversion algorithm. The revised procedure is presented in the Appendix.

2. EMABIC CODES AND STRUCTURE

2.1 Overview

EMABIC was conceived as a general diagnostic tool for the retrieval of gas and particle properties of low-visibility propellant, tactical rocket motor plumes from experimental E/A and scattering measurements made on plumes. The program name DPREP1 has been reserved for a code that would process such data (e.g., smoothing, extrapolation, scaling) and produce the transverse data profiles $\bar{N}(z)$, $\bar{\tau}(z)$, $N_p(z)$, $\tau_p(z)$, and $f(z,\theta)$ required by the inversion procedures. In lieu of experimental data, a set of transverse data simulation codes has been envisioned consisting of the programs DPREP2, EAPROF, and NOISE. At present, only the programs DPREP2 and EAPROF have been completed. DPREP2 is a data preparation code that generates a file of radial plume properties (e.g., temperature, concentrations, band model parameters). Its primary use is the preparation of input for the actual transverse profile generation code EAPROF. NOISE is a planned program that will add random errors to the profiles generated by EAPROF in order to study error propagation in the retrieval programs. The general data flow for the preparation of input data for retrieval is shown in Fig. 1.

The inversion programs of EMABIC consist of three programs (PARIC1, PARIC2, and PARIC3) for the retrieval of plume particle properties and one code (GASIC) for the retrieval of plume gas properties. The general flow of data for retrieval is illustrated in Fig. 2. Briefly, PARIC1 retrieves the volume extinction cross section $\gamma(r)$ from the transverse particle-only transmittance profile $\tau_p(z)$; PARIC2 retrieves the volume absorption cross section $\alpha(r)$, the volume scattering cross section $\beta(r)$, and the scattering phase function $p(r,\theta)$ from the laser scattering efficiency function $f(z,\theta)$; PARIC3

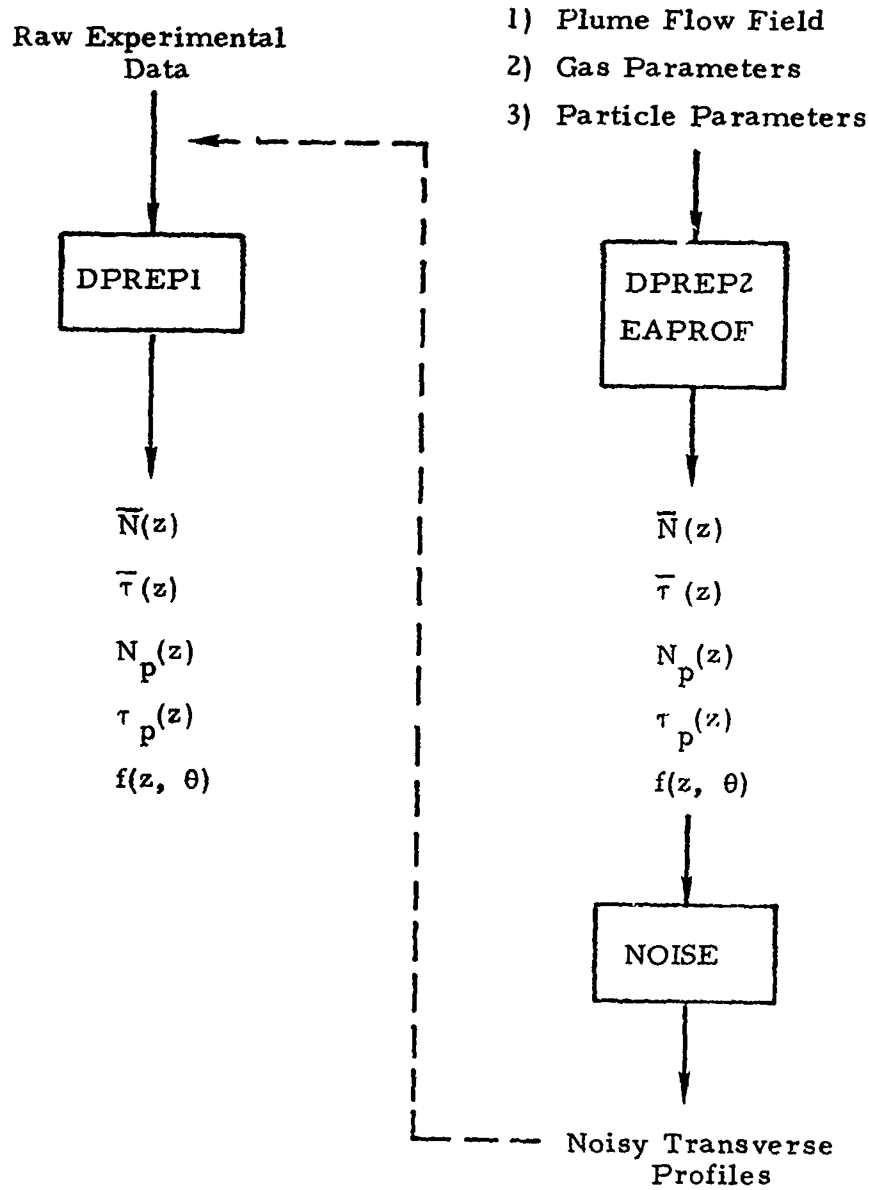

Fig. 1. EMABIC Radiation Prediction Flow Diagram

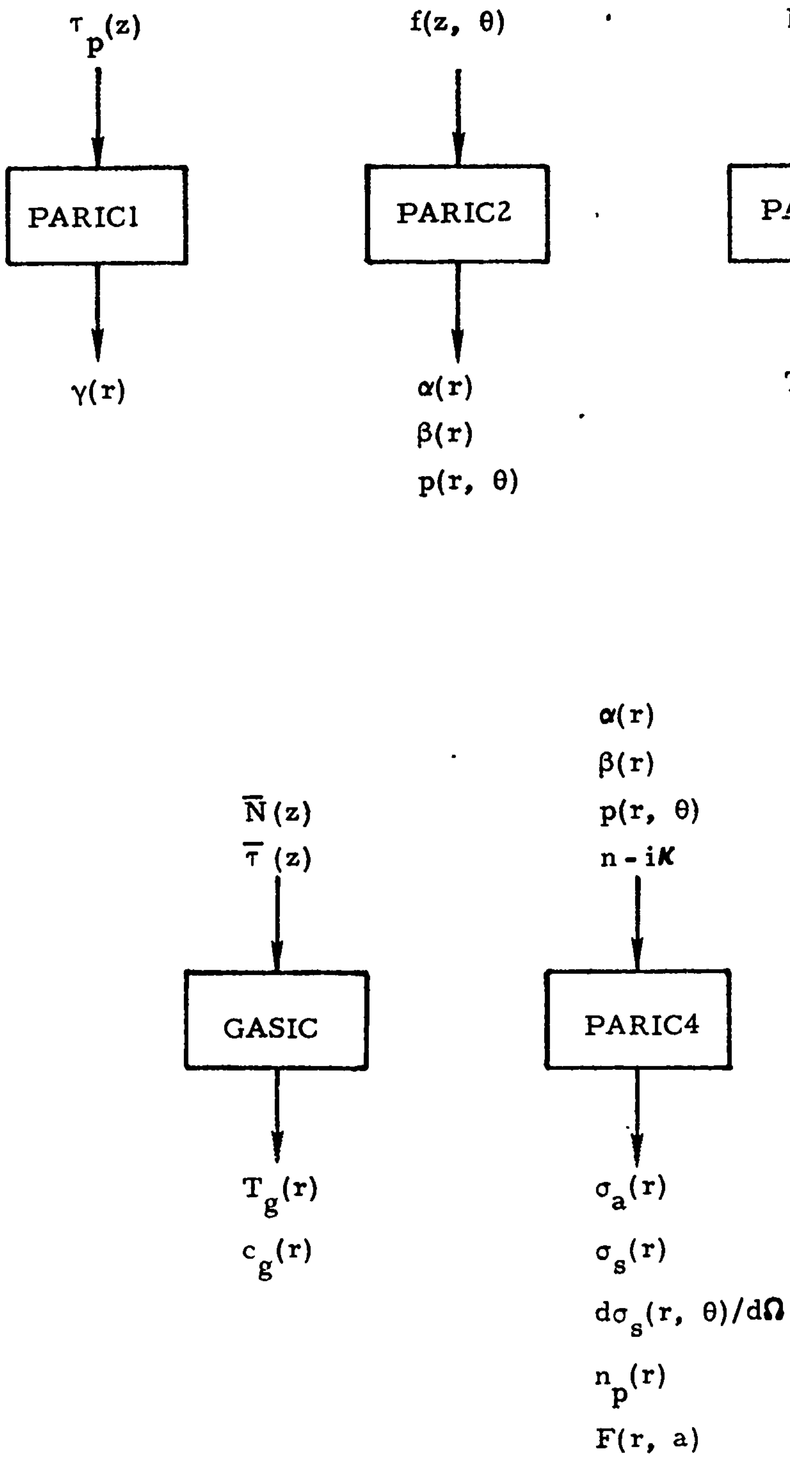

Fig. 2. EMABIC Retrieval Flow Diagram

retrieves the particle temperature profile $T_p(r)$ from the particle-only radiance profile $N_p(z)$; and GASIC retrieves the gas temperature $T_g(r)$ and concentration $c_g(r)$ from the coupled gas-plus-particle radiance $\tilde{N}(z)$ and transmittance $\bar{\tau}(z)$.

A fourth particle retrieval code (PARIC4) is planned that would allow the retrieval of the intrinsic particle scattering cross sections $\sigma_a(r)$, $\sigma_s(r)$ and $d\sigma_s(r)/d\Omega$ from $\alpha(r)$, $\beta(r)$, and $p(r,\theta)$, respectively, as well as the retrieval of the absolute particle loading $n_p(r)$ and particle size distribution $F(r,a)$ (a = particle radius). Retrieval with PARIC1, PARIC2, PARIC3, and GASIC requires no assumptions on the index of refraction or shape of the plume particles. Implementation of PARIC4, however, would. Its development will assume that the particles are homogeneous spheres (so that standard Mie scattering theory can be used) and that the particle composition (and thus the index of refraction $m = n - i\kappa$) is known.

Data flow within EMABIC is handled with three random access files. The file generated by DPREP2 is called the "a priori radial data file" and is used primarily as input for the transverse profile generation program EAPROF. The output of EAPROF is written to a file called the "transverse data file" (DPREP1 would generate this same file) and is the basic input for the retrieval codes. Finally, the retrieval codes write their results to a file called the "retrieved radial data file." A summary of the contents of these files, and the codes that write to and read from them, is given in Table 1. Detailed discussions of the files are given in the following sections.

In general, the application of the inversion codes to actual data must be made in the order PARIC1, PARIC2, PARIC3, and GASIC because the retrieved result of each code is required by the subsequent codes. (PARIC4 could follow immediately after PARIC3; the results of GASIC would not be required.) For

Table 1. Summary of Data Transfer Files

Tape Number	File Name	Description	Written to by (Code/Routine)	Read from by (Code/Routine)
TAPE2	*A priori* radial data	Contains radial plume pTc data for emission profile calculations or "assumed" plume properties for inversion calculations	DPREP2	EAPROF/INPUT1 PARIC2/INPUT3 PARIC3/INPUT4 GASIC/INPUT5
TAPE3	Transverse data	Contains transverse plume emission/absorption/scattering data for inversion calculations	EAPROF/OUTPUT1 DPREP1*	PARIC1/INPUT2 PARIC2/INPUT3 PARIC3/INPUT4 GASIC/INPUT5
TAPE4	Retrieved radial data	Contains radial plume pTc data retrieved from the transverse plume data with the particle and gas inversion codes	PARIC1/OUTPUT2 PARIC2/OUTPUT3 PARIC3/OUTPUT4 GASIC/OUTPUT5	PARIC2/INPUT3 PARIC3/INPUT4 GASIC/INPUT5

*Not used in current version of EMABIC.

example, $\gamma(r)$, which is obtained with PARIC1, is stored on the "retrieved radial data" file and used by all of the subsequent codes. Anticipating that no experiment is likely to be performed in the near future that would measure all of the data required for a full implementation of the EMABIC retrieval capability, provisions have been made for operating each code with "assumed" results from a previous code. These provisions are made by allowing each code to obtain its required radial data from the "a priori radial data file" rather than from the "retrieved radial data file." These "assumed" values are placed on the "a priori radial data file" with DPREP2.

In its present form, EMABIC consists of the six main programs DPREP2, EAPROF, PARIC1, PARIC2, PARIC3, and GASIC, 45 subprogram units, and three implied data files. The subprograms form a common base from which the six main programs call (either directly or indirectly) as required. Some subprograms are called by only one main program, but most are called by more than one. Brief descriptions of these subprograms are given in the following sections as they arise in the discussions of the six main programs. As aids to understanding the structure of EMABIC, the subprograms are grouped according to general computational function in Table 2, and the subroutines actually called by each of the main programs are indicated in the routine-calling flow diagrams of Figs. 3 through 8. (These should not be interpreted as detailed calculation flow diagrams.)

2.2 Radiation Prediction Codes

The new codes DPREP2 and EAPROF replace the older code EAPROF (Refs. 1 and 2) and add the capability for general radial variation of particle scattering parameters and the calculation of the laser scattering efficiency function. These codes compute the transverse profiles of infrared emission and extinction and laser scattering for an axisymmetric, axially uniform,

Table 2. Grouping of Subroutines by Computational Function.

Data Preparation Routines

BPARAM ZAFIT NKPARAM*
KDPARAM ZONEFIT
ANGLFIT

Input/Output Routines

INPUT1	OUTPUT1
INPUT2	OUTPUT2
INPUT3	OUTPUT3
INPUT4	OUTPUT4
INPUT5	OUTPUT5

Convergence Testing Routines

CONVRG1
CONVRG2
CONVRG3

Radiation Routines

SRCFUNC	JSCAT	PLANCK	YCGL	F	**
KERNEL	GSCAT	YDRL	YDRL	G	
PROFILE	FSCAT		YLSL	WMIX	
TRANS			YCGD		
QSCAT			YMLD		
RADNCE			YLSD		
SLOS			YMIX		

Inversion Routines

ABEL
CTSOLVE

Quadrature Routines

SOLID
QUAD
COEFF

*Not used in current version of EMABIC.

**These ten subroutines comprise the subroutine set RADECK.

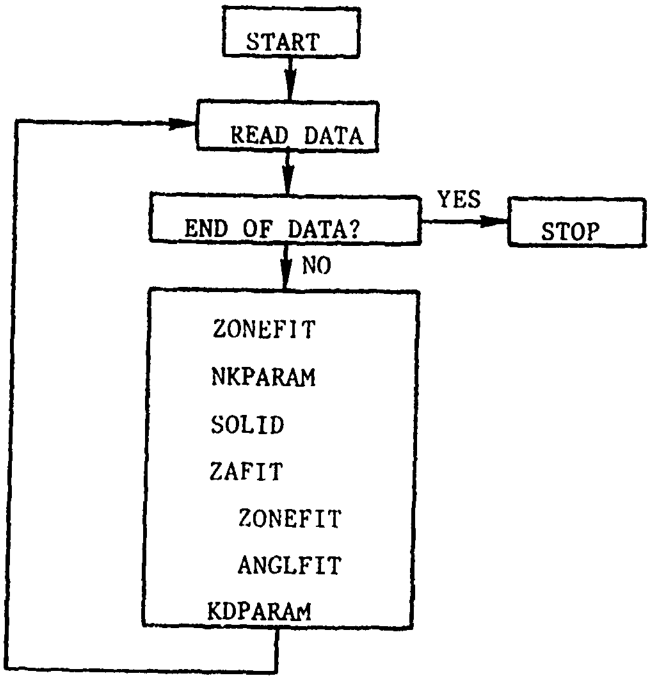

Fig. 3. Subroutine-Calling Flow Diagram for DPREP2. Degree of Indentation of subroutine name indicates the level at which it is called.

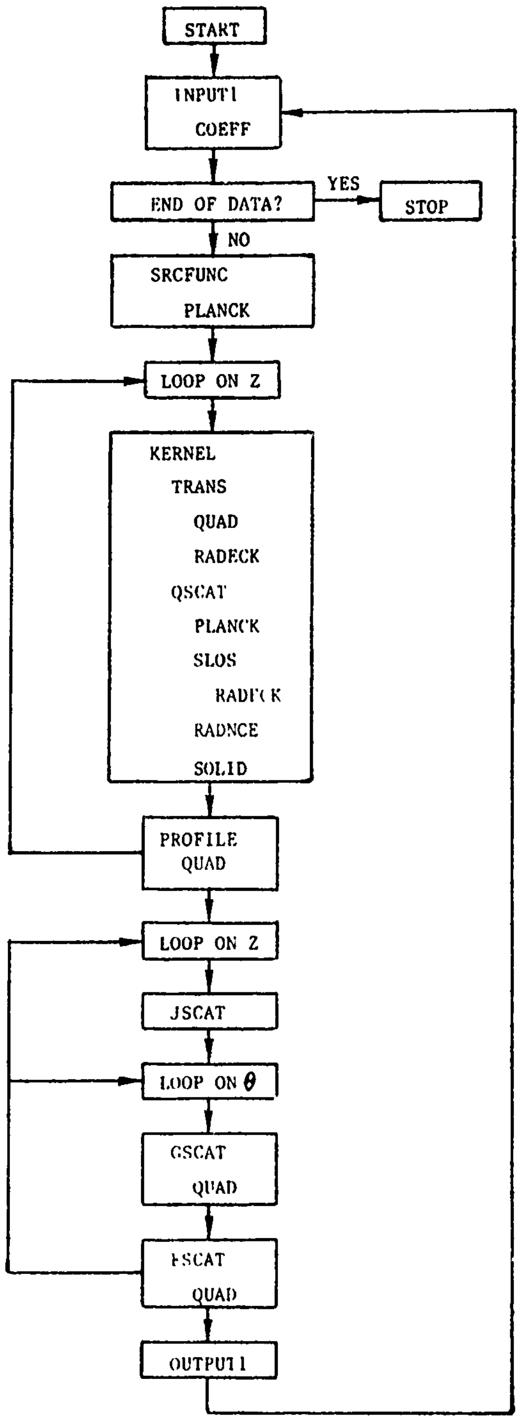

Fig. 4. Subroutine-Calling Flow Diagram for EAPROF. Degree of indentation of subroutine name indicates the level at which it is called.

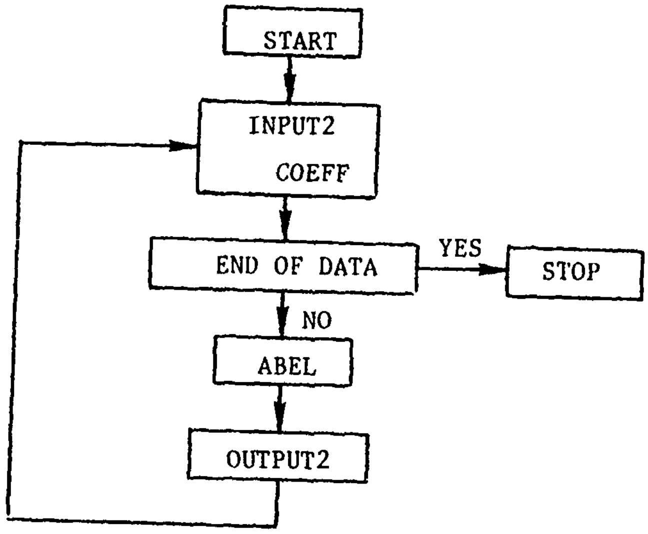

Fig. 5. Subroutine-Calling Flow Diagram for PARIC1. Degree of indentation of subroutine name indicates the level at which it is called.

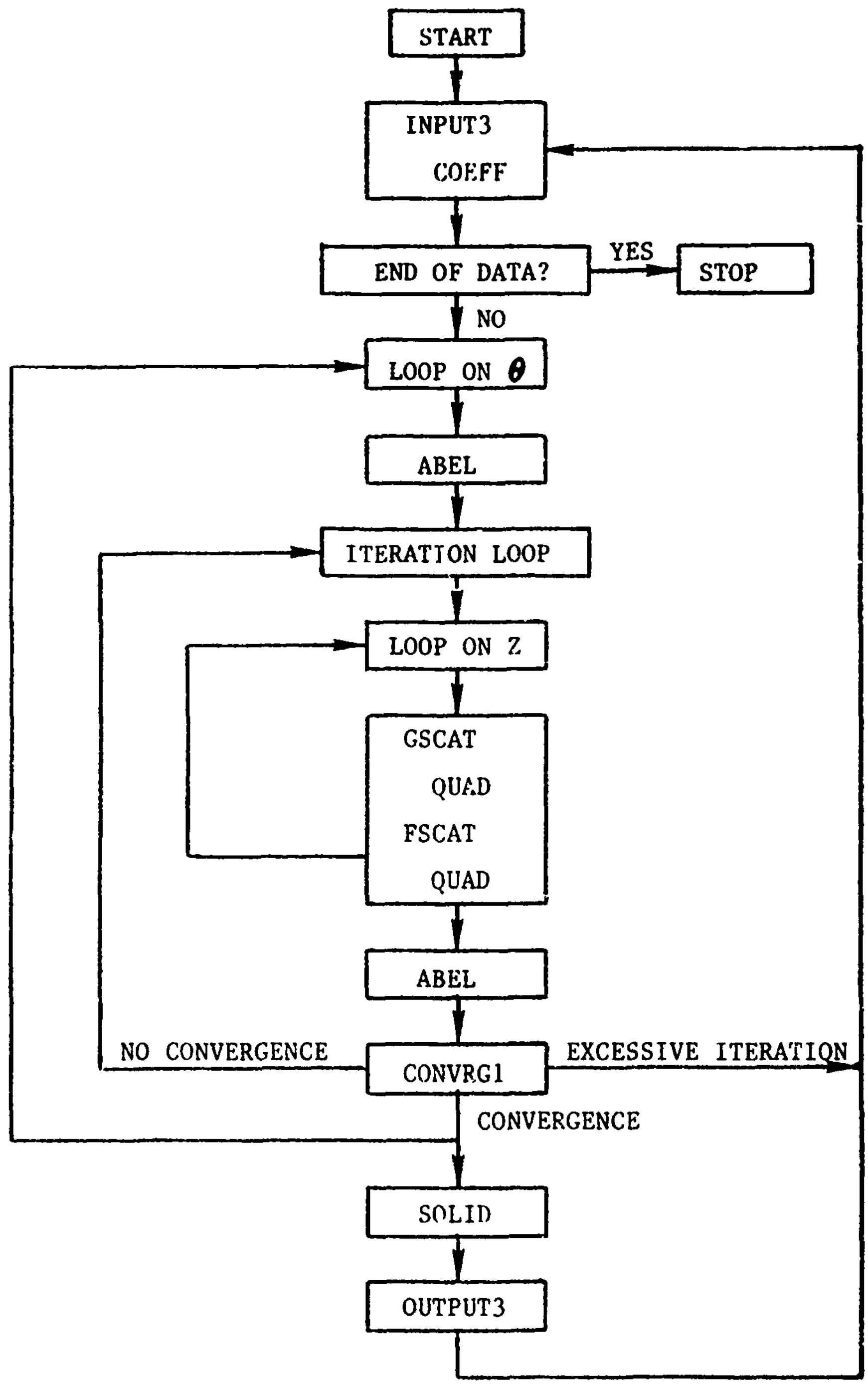

Fig. 6. Subroutine-Calling Flow Diagram for PARIC2. Degree of indentation of subroutine name indicates the level at which it is called.

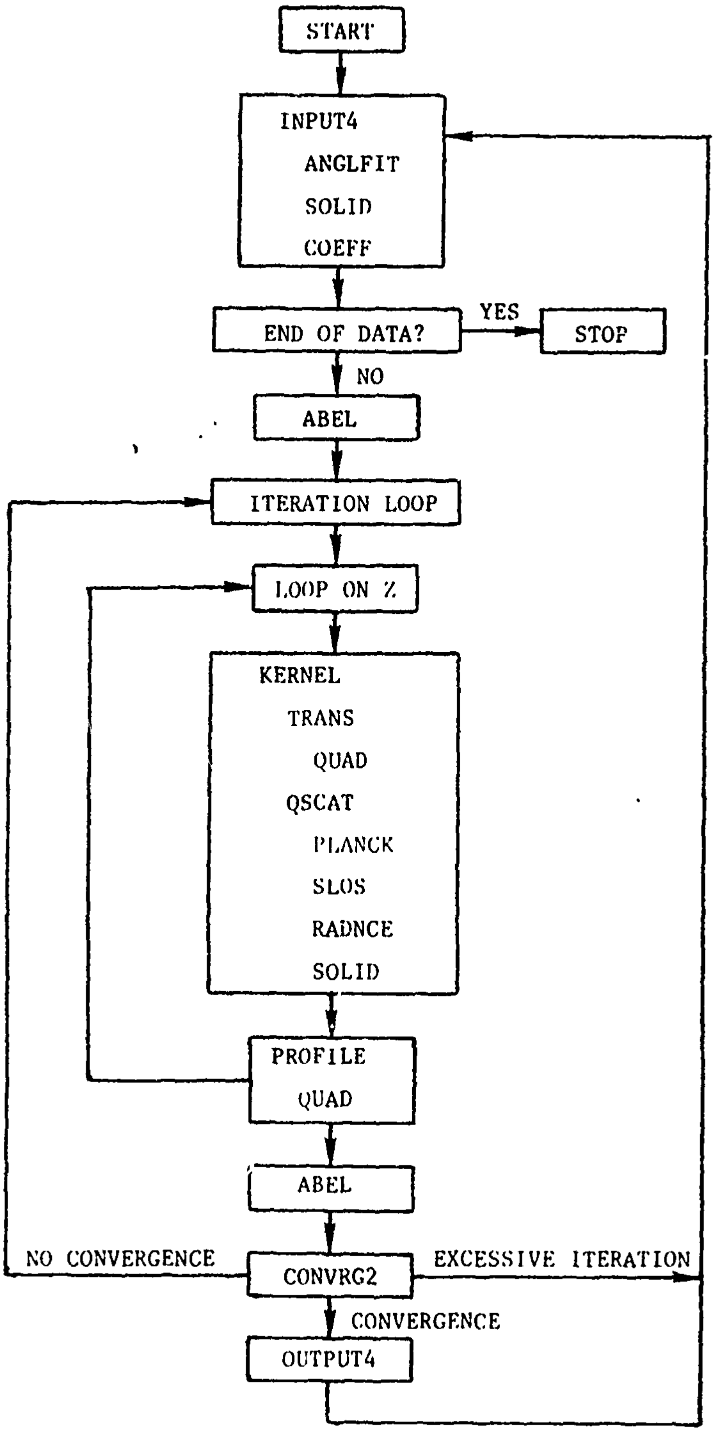

Fig. 7. Subroutine-Calling Flow Diagram for PARIC3. Degree of indentation of subroutine name indicates the level at which it is called.

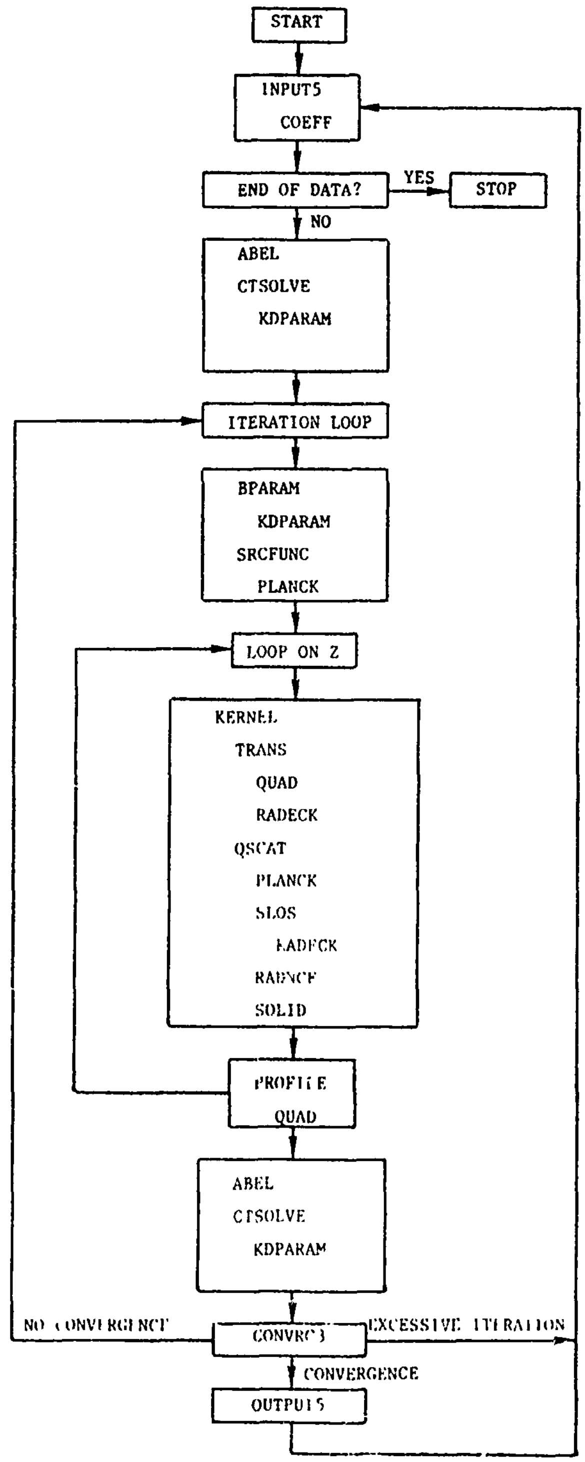

Fig. 8. Subroutine-Calling Flow Diagram for GASIC. Degree of indentation of subroutine name indicates the level at which it is called.

cylindrical plume from radial profiles of gas temperature, pressure, and concentration and particle temperature and number density. The radiation model treats gas radiation transfer with band model methods and particle radiation transfer with the single-scattering approximation. The radiation model correctly couples the gas and particle components into a single emitting, absorbing, and scattering medium. The program treats just one gas species and one particle species at a time.

The gas band model is the Malkmus statistical model and employs either the Curtis-Godson (CG) or derivative (DR) approximations to handle the inhomogeneity and nonisothermality of the plume. Lorentz, Doppler, or Voigt gas spectral line profiles may be used.

The single-scattering geometry used for particle radiation transport is shown in Fig. 9. The s-axis is the primary line of sight (LOS). The LOS shown in Fig. 9 is the one that goes through the full plume diameter. As the LOS is scanned out across the lateral extent of the plume, it cuts progressively shorter chords of the cylindrical plume. The σ-axis is the scattering LOS. It is described by the value of s where it branches off the primary LOS and by the scattering angle θ and the azimuthal angle ϕ. The single-scattering approximation includes radiation emitted along the primary LOS and radiation that has been scattered once from the scattering LOS into the primary LOS. If the scattering LOS terminates on the nozzle exit plane, motor radiation scattered into the primary LOS is also included. The exit plane is modeled as a solid disc with uniform temperature and emissivity.

Extinction of external radiation shone through the plume is assumed to be caused by gas absorption, particle absorption, and particle outscattering. The single-scattering approximation does not allow inscattering to contribute to extinction calculations.

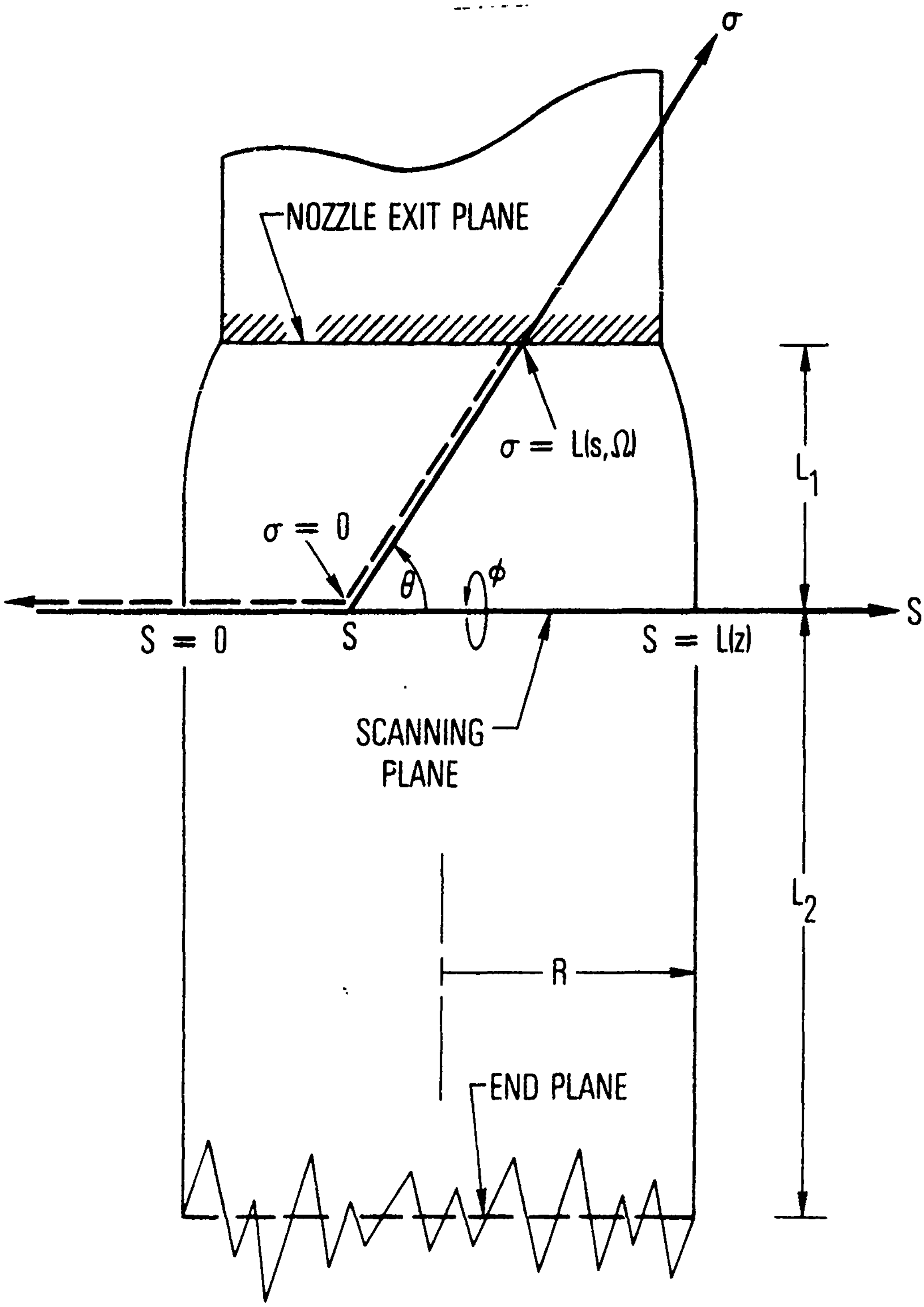

Fig. 9. Plume Scattering Geometry

2.2.1 Program DPREP2

The purpose of DPREP2 is to prepare a file of radial plume data. Various data defined on arbitrary radial and angle grids are read in by DPREP2 and fitted to a desired radial grid, scattering angle grid, and laser scattering angle grid. The radial input data are the temperature and concentration profiles for the gas and particle species of interest, the band model parameters for the gas, and the scattering cross sections for the particles. The input grid data are the number of equal-size radial zones to which the radial data are fitted, the solid angle integration scattering angle grid θ, and the laser scattering angle grid θ_L. Required formats for these data are discussed in Section 3.2.

The first step of data preparation is the fitting of the input temperature and concentration data for both the gas and particles to the equal-zone radial grid. This is performed by linear interpolation in the subroutine ZONEFIT. With the gas temperature, pressure, and concentration that obtain on the grid, quadratic interpolation is made on the input band model parameters in subroutine KDPARAM in order to obtain the parameters on the radial grid. Angle independent particle scattering parameters are fitted to the radial grid with ZONEFIT and the input table of scattering parameters. The angle dependent scattering parameter (the differential scattering cross section) is fitted to the radial grid and the integration and laser scattering angle grids (θ and θ_L) in subroutine ZAFIT. This subroutine calls ZONEFIT for the required radial interpolation and ANGLFIT for the required angle interpolation. The latter interpolation is linear. After fitting to the grids θ and θ_L, the differential scattering cross sections, on each grid, are renormalized to 4π by use of subroutine SOLID. This subroutine calculates the integral over 4π steradians of an arbitrary function $f(\theta)$ defined between

$\theta = o$ and π.

After further straightforward calculations, the a priori radial file is written. The format of this file is shown in Table 3.

2.2.2 Program EAPROF

Program EAPROF calculates transverse E/A and laser scattering efficiency functions. Its main source of input data is the a priori radial data file. These data, as well as additional data in card form (see Section 3.3 for data preparation) are read by INPUT1. The basic E/A calculations are performed in a loop over transverse position that calls the three subroutines SRCFUNC, KERNEL, and PROFILE. The first subroutine computes the radial source function J(r) as defined in Table 1 of Ref. 5.* The second computes the kernel functions G(z,r) defined in the same reference. The third subroutine computes the transverse profiles F(z) defined by (see Eq. 29, Ref. 5)

$$F(z) = 2 \int_z^R J(r)\, G(z,r) \frac{r dr}{(r^2 - z^2)^{1/2}} .$$

The numerical quadrature of this integral is performed in subroutine QUAD (with quadrature coefficients computed by COEFF) and is described in detail in Ref. 3.

Calculation of the source functions J(r) for emission requires an evaluation of the Planck (blackbody) radiation function. This is performed in the subprogram PLANCK.

Calculation of the kernel functions G(z,r) requires the prior calculation of the gas transmittance $\overline{\tau}^{\pm}(z,r)$, the gas derivative functions $y^{\pm}(z,r)$,

---

* For a full appreciation of this entire section, it is recommended that Section 2 and Table 1 of Ref. 5 be close at hand.

Table 3. A Priori Radial Data File.

Index	Variable	Length	Description
1	TITLE	7	DPREP2 job run title
2	GNAME	1	Gas species identification name
3	PNAME	1	Particle species identification name
4	AGRDID	1	Scattering angle grid identification name
5	LGRDID	1	Laser scattering grid identification name
6	GDTAID	1	Gas data identification name
7	PDTAID	1	Particle data identification name
8	GPRMID	1	Gas band model parameters identification name
9	PPRMID1	1*	Particle index of refraction identification name
10	PPRMID2	1	Particle scattering parameters identification name
11	ν	1	Wavenumber of gas parameters (cm^{-1})
12	ν_p	1	Wavenumber of particle parameters (cm^{-1})
13	N_r	1	Number of radial grid points
14	N_{θ_s}	1	Number of scattering angle grid points
15	N_{θ_L}	1	Number of laser scattering angle grid points
16	r	51	Radial grid points (cm)
17	P_g	51	Total gas pressure (atm)
18	T_g	51	Gas temperature (K)
19	T_p	51	Particle temperature (K)
20	c_g	51	Gas concentration (mole fraction)
21	n_p	51	Particle concentration (cm^{-3})
22	k	51	Absorption band model parameter (cm^{-1}/atm)
23	$1/\bar{\delta}$	51	Line density band model parameter ($1/cm^{-1}$)
24	$\bar{\gamma}_L$	51	Pressure broadened line width (cm^{-1})
25	$\bar{\gamma}_D$	51	Doppler broadened line width (cm^{-1})
26	n	51*	Particle index of refraction (real part)
27	κ	51*	Particle index of refraction (imaginary part)
28	σ_a	51	Particle absorption cross section (cm^2)
29	σ_s	51	Particle scattering cross section (cm^2)
30	α	51	Particle volume absorption cross section (cm^{-1})
31	β	51	Particle volume scattering cross section (cm^{-1})
32	θ_s	37	Scattering angle grid (deg)
33	$d\sigma_s/d\Omega$	1887	Particle differential scattering cross section (cm^2/sr) on θ_s grid
34	p_s	1887	Particle scattering phase function (sr^{-1}) on θ_s grid
35	θ_L	37	Laser scattering angle grid (deg)
36	$d\sigma_L/d\Omega$	1887	Particle differential scattering cross section (cm^2/sr) on θ_L grid
37	p_L	1887	Particle scattering phase function (sr^{-1}) on θ_L grid

*Not used in current version of EMABIC.

the particle transmittances $\tau_\alpha^{\pm}(z,r)$ and $\tau_\beta^{\pm}(z,r)$, and the scattering source functions $Q_S^{\pm}(z,r)$. All except $Q_S^{\pm}(z,r)$ are computed in subroutine TRANS. This subroutine employs the band model procedures described in Ref. 3 for computing $\bar{\tau}^{\pm}(z,r)$ and $y^{\pm}(z,r)$. In addition to calls to QUAD for computing path averaged band model parameters, any of the secondary radiation routines of the set RADECK (i.e., YCGL, YDRL, YLSL, YCGD, YMLD, YLSD, YMIX, F, G, or WMIX) may be called, depending on the gas spectral lineshape and path nonuniformity approximation specified as part of the EAPROF input. These routines are described in Ref. 3 and works referenced therein. The particle transmittance functions $\tau_\alpha^{\pm}(z,r)$ and $\tau_\beta^{\pm}(z,r)$ are calculated directly in TRANS.

The scattering source functions $Q_S^{\pm}(z,r)$ are computed in subroutine QSCAT. This subroutine calculates the integral over 4π steradians ($o < \theta < \pi$, $o < \phi < 2\pi$) of radiation impinging on a specified point on an observation line of sight. The integration over θ is performed in subroutine SOLID. The actual radiance impinging at the point from direction (θ,ϕ) is calculated in subroutine RADNCE. The gas and particle properties along the scattering line of sight required for the radiance calculation are set up in subroutine SLOS. The gas band model calculations of RADNCE and SLOS parallel the procedures used in TRANS. The calculation of $Q_S^{\pm}(z,r)$ includes the contribution of exit plane radiation.

The calculation of the laser scattering efficiency function $f(z,\theta)$ takes place in a double loop over laser scattering angles θ_L and transverse position z that calls the subroutines JSCAT, GSCAT, and FSCAT. The first subroutine computes the scattering source function $J(r,\theta)$ (Eq. 43 of Ref. 5), the second computes the kernal function $G(z,r,\theta)$ (Eq. 44 of Ref. 5), and the third computes the scattering efficiency function (Eq. 42 of Ref. 5)

$$f(z,\theta) = 2 \int_z^R J(r,\theta)\ G(z,r,\theta)\ \frac{r\,dr}{(r^2 - z^2)^{1/2}}.$$

As in the subroutine PROFILE, this integral is computed in subroutine QUAD.

When the computation of the E/A and scattering functions are complete, the results are listed and written to the transverse data file in subroutine OUTPUT1. The format of the output file is given in Table 4.

2.3 Particle Property Retrieval Codes

2.3.1 Program PARIC1

Program PARIC1 retrieves the radial volume extinction profile $\gamma(r)$ from the transverse particle-only transmittance profile $\tau_p(z)$. The transverse data and other input data are read by subroutine INPUT2 (see Section 3.4 for data preparation). Retrieval is accomplished by Abel inversion (Eq. 48, Ref. 5) of $\tau_p(z)$ in the subroutine ABEL (see Appendix). The result for $\gamma(r)$ is written to the retrieved radial data file by OUTPUT2. The format of this file is given in Table 5.

2.3.2 Program PARIC2

Program PARIC2 retrieves the radial volume scattering cross section $\beta(r)$, volume absorption cross section $\alpha(r)$, and scattering phase function $p(r,\theta)$ from the transverse laser scattering efficiency function $f(z,\theta)$. The transverse data and other input data are read in by subroutine INPUT3 (see Section 3.5 for data preparation). Retrieval is made by iterative Abel inversion for the scattering source function $J(r,\theta)$ (Eq. 49, Ref. 5). A first approximation, $J'(r,\theta)$, is made for $J(r,\theta)$ by simple Abel inversion of $f(z,\theta)$. This first approximation is then used to compute a new function $f'(z,\theta)$. This computation is made exactly as is done in EAPROF (Section 2.2.2) except that subroutine JSCAT is not called. An Abel inversion of the difference between f and f' is made in order to get a correction ΔJ to

Table 4. Transverse Data File.

Index	Variable	Length	Description
1	TITLE	7	DPREP1 or EAPROF job run title
2	GNAME	1	Gas species identification name
3	PNAME	1	Particle species identification name
4	ν	1	Wavenumber of gas and particle data (cm^{-1})
5	ν_p	1	Wavenumber of particle-only data (cm^{-1})
6	N_z	1	Number of transverse grid points for E/A data
7	N_{z_L}	1	Number of transverse grid points for laser scattering data
8	N_{θ_L}	1	Number of laser scattering angle grid points
9	z	51	Transverse grid points (cm)
10	$\bar{N}_g$	51	Gas-only radiance (W/cm^2-sr-cm^{-1})
11	$\bar{\tau}_g$	51	Gas-only transmittance
12	N_p	51	Particle-only radiance (W/cm^2-sr-cm^{-1})
13	τ_p	51	Particle-only transmittance
14	$\bar{N}$	51	Total radiance (W/cm^2-sr-cm^{-1})
15	$\bar{\tau}$	51	Total transmittance
16	p_g	51	Total gas pressure (atm)
17	θ_L	31	Laser scattering angle grid (deg)
18	f	1887	Laser scattering efficiency (sr^{-1})

Table 5. Retrieved Radial Data File.

Index	Variable	Length	Description
1	TITLE1	7	Transverse data file TITLE
2	TITLE2	7	PARIC1 job run title
3	TITLE3	7	PARIC2 job run title
4	TITLE4	7	PARCI3 job run title
5	TITLE5	7	GASIC job run title
6	TITLE6	7*	PARIC4 job run title
7	GNAME	1	Same as *a priori* radial data file
8	PNAME	1	Same as *a priori* radial data file
9	N_r	1	Same as *a priori* radial data file
10	N_{θ_s}	1	Same as *a priori* radial data file
11	N_{θ_L}	1	Same as *a priori* radial data file
12	N_D	1*	Number of particle sizes
13	r	51	Same as *a priori* radial data file
14	T_g	51	Same as *a priori* radial data file
15	T_p	51	Same as *a priori* radial data file
16	c_g	51	Same as *a priori* radial data file
17	n_p	51*	Same as *a priori* radial data file
18	α	51	Same as *a priori* radial data file
19	β	51	Same as *a priori* radial data file
20	γ	51	Volume extinction cross section (cm^{-1})
21	σ_a	51*	Same as *a priori* radial data file
22	σ_s	51*	Same as *a priori* radial data file
23	θ_s	37	Same as *a priori* radial data file
24	$d\sigma_s/d\Omega$	1887*	Same as *a priori* radial data file
25	p_s	1887	Same as *a priori* radial data file
26	θ_L	37	Same as *a priori* radial data file
27	$d\sigma_L/d\Omega$	1887*	Same as *a priori* radial data file
28	p_L	1887	Same as *a priori* radial data file
29	a	30*	Particle size grid (μm)
30	F	1530*	Particle distribution (μm^{-1})

*Not used in current version of EMABIC

$J'(r,\theta)$. If ΔJ is small enough (as determined by an input convergence criterion and subroutine CONVRG1), iteration ceases. Otherwise, it is continued with J' replaced by $J' + \Delta J$. When $J(r,\theta)$ is determined to within the convergence criterion, it is integrated over 4π steredians in subroutine SOLID to obtain $\beta(r)$ (Eq. 50, Ref. 5). Then, $\alpha(r) = \gamma(r)\ \beta(r)$ and $p(r,\theta) = 4\pi\ J(r,\theta)/\beta(r)$. The results are written to the retrieved radial data file in OUTPUT3.

2.3.3 Program PARIC3

Program PARIC3 retrieves the radial particle temperature profile $T_p(r)$ from the transverse particle-only radiance $N_p(z)$. The transverse and other input data are read in by subroutine INPUT4 (see Section 3.6 for data preparation). In addition to reading the data, INPUT4 also fits and renormalizes the scattering phase function obtained in PARIC3 (which is defined on the laser scattering grid θ_L) to an integration scattering angle grid θ. The fitting and renormalization are performed with the subroutines ANGLFIT and SOLID. The actual retrieval of $T_p(r)$ occurs by iterative Abel inversion of $N_p(z)$ to obtain the source function $J(r) = \alpha(r)\ B_p(r)$ (Eq. 55, Ref. 5). The iterative computation of $N_p(z)$ is performed as in EAPROF except that the source function subroutine SRCFUNC is not used. Convergence of the iteration is tested in CONVRG2. When $J(r)$ [and thus $B_p(r)$] is determined to within the convergence criterion, a straightforward inversion of the Planck function is made for $T_p(r)$. The result is written to the retrieved radial data file in OUTPUT4.

2.4 Gas Property Retrieval Code (Program GASIC)

Program GASIC retrieves the radial gas temperature $T_g(r)$ and concentration $c_g(r)$ from the transverse total radiance $\bar{N}(z)$ and transmittance $\bar{\tau}(z)$ profiles. The transverse and other data are read in by subroutine INPUT5 (see

Section 3.7 for data preparation). Retrieval is made by simultaneous iterative Abel inversion of the total transverse radiance $\bar{N}(z)$ and transmittance $\bar{\tau}(z)$ profiles to obtain the source functions $\bar{\kappa}(r)$ $B_g(r)$ and $\bar{\kappa}(r)$ (Eq. 56, Ref. 5). At each iteration, the subroutine CTSOLVE is used to retrieve $T_g(r)$ and $c_g(r)$ from the current source functions, and convergence is tested in subroutine CONVRG3. The iterative computation of $\bar{N}(z)$ and $\bar{\tau}(z)$ is performed as in EAPROF except that the source function subroutine SRCFUNC is not called. With each iteration result for T_g and c_g, new values for band model profiles on the radial grid are determined in subroutine BPARAM (which calls KDPARAM). When T_g and c_g are determined to within the convergence criteria, the results are written to the retrieved radial data file in OUTPUT5.

The inversion procedure described above is for the full retrieval scheme developed in Ref. 5. Slight modifications are made for the gas-only and first-order, off-band retrieval approximations options. In the first approximation, a gas-only inversion is made on $\bar{N}(z)$ and $\bar{\tau}(z)$ as though they were for a purely gaseous plume. Particle effects are totally neglected. In the second, a gas-only inversion is made on $\bar{N}(z) - N_p(z)$ and $\bar{\tau}(z)/\tau_p(z)$.

3. PREPARATION OF INPUT DATA

3.1 General Instructions

A computational run of each of the programs DPREP2, EAPROF, PARIC1, PARIC2, PARIC3, OR GASIC requires a set of program control cards to specify the mode of computation and to supply input data. Some program control cards simply specify a computation mode, some specify a computation mode and supply data, while others signal the codes that blocks of auxiliary data are now to be read. Each type of control card contains an alphanumeric name in the first ten card columns. These names must be spelled correctly and must be left-justified. If data are specified on a program control card, they must be entered in accordance with the format specification indicated in the detailed description of each card given below. All fields of the program control cards are 10 columns wide. In general, integer and alphanumeric data must be right-justified in their fields. Noninteger numerical data may be entered in either F or E formats (with decimal point and, for the latter, the exponential symbol E). E-formatted data must be right-justified in their fields. These same rules apply to data entered on auxiliary card decks.

Each of the programs has a multiple run feature. After all of the data required for a run have been entered by way of control cards, auxiliary card decks, or attached data files, execution is initiated with the RUN control card. When the computation and results listing are completed, the program continues to read program control cards until a new RUN card is encountered. A new computation is then begun for all of the conditions and data of the first run except those that have been changed by the intervening program control cards and auxiliary data decks. This process is repeated until an end-of-file card is encountered. With this feature, a large number of related runs can be made with one job submission. All of the codes write their results to a data file (TAPE2, TAPE3, OR TAPE4). If multiple runs are made for any of the codes with one job submission, only the results of the last run are saved on the appropriate file.

In general, all program control cards that call for the read in of data from card decks or attached files have a print option for listing the data. If the variable PRINT (format A10) on these cards has the alphanumeric value PRINT, the data will be listed. The description of this option is not repeated in the following sections.

A control card common to all of the program is the title card. Its name is TITLE, and columns 11 through 80 of this card may be used for any run identification title desired. Use of the card is optional. Again, the description of this card is not repeated in the following sections.

Other than the requirement that all required data be specified before a RUN card is encountered and that auxiliary data on card decks immediately follow the control card that calls for them, the program control cards may for the most part, be arranged in any order. Exceptions are specifically mentioned in the detailed descriptions that follow.

Great care should be taken in the preparation of input data since very few checks of data consistency, setting of default values, or issuance of error messages are provided. A general feature of data preparation is that, if particular data on a card are not required, they need not be specified. If none of the data on a control card is needed, that card need not be included.

3.2 DPREP2 Data Preparation

Program DPREP2 prepares radial and angular data for use by the E/A prediction code EAPROF. The types of control cards required for a run of program DPREP2 are illustrated in Fig. 10. A description of each type follows.

Card	1–10	11–20	21–30	31–40	41–80
1	TITLE				
2	DATA	NZONES	GCOL	PCOL	
3	AGRID	PRINT			
4	LGRID	PRINT			
5	GDATA	PRINT			
6	PDATA	PRINT			
7	GPARAM	PRINT			
8	PPRAM2	PRINT			
9	LIST				
10	NOLIST				
11	RUN				

Fig. 10. DPREP2 Program Control Card Formats

1. Title Card.

2. Miscellaneous Data Card. The card name is DATA. NZONES (format I10) is the number of equal-size radial zones into which the plume is divided. The maximum allowed value is 50. (NZONES also defines the number of transverse zones used in EAPROF.) GCOL and PCOL (each format I10) identify, respectively, which gas and particle species are to be selected from the auxiliary input decks of radial gas and particle data (Figs. 12 and 13, respectively). GCOL = 1, 2, 3, or 4; PCOL = 1, 2, or 3.

3. Scattering Angle Integration Grid Card. The card name is AGRID. This card calls for the read in of the scattering angle grid that EAPROF will use for integration over solid angles. The required deck structure for these data is shown in Fig. 11.

4. Laser Scattering Angle Grid Card. The card name is LGRID. This card calls for the read in of the angle grid for which EAPROF will compute the laser scattering efficiency function $f(z,\theta)$. The required deck structure for the data is shown in Fig. 11.

5. Radial Gas Data Card. The card name is GDATA. This card calls for the read in of radial gas data. The required deck structure for the data is shown in Fig. 12.

6. Radial Particle Data Card. The card name is PDATA. This card calls for the read in of radial particle data. The required deck structure for the data is shown in Fig. 13.

7. Gas Band Model Parameters Card. The card name is GPARAM. This card calls for the read in of band model parameters for the gas species of interest. It is the user's responsibility to ensure that the parameters read in are consistent with the gas species

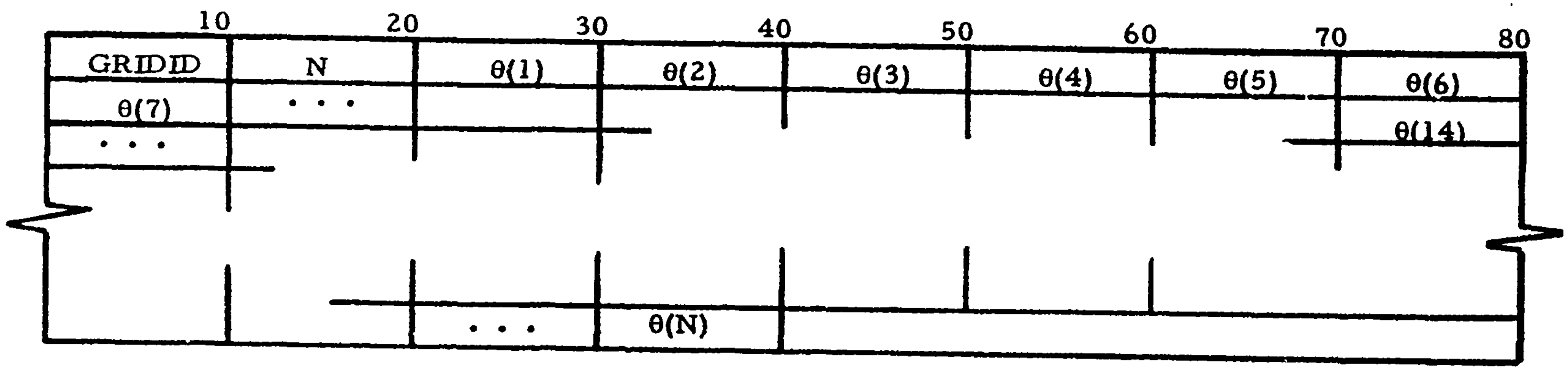

Fig. 11. Input Card File Structure for Scattering Angle Integration Grid and Laser Scattering Angle Grid

LEGEND

All field formats are E10 or F10 except the GRIDID field which is A10 and the N field which is I10.

GRIDID	Angle grid identification name.
N	Number of scattering angles ($N \le 37$).
$\theta(i)$	Scattering angle array (deg). $0 \equiv \theta(1) < \theta(2) < \ldots < \theta(N) \le 180^\circ$.

10	20	30	40	50	60	70	80
GDTAID	N	R	GNAME1	GNAME2	GNAME3	GNAME4	
r(1)	p(1)	T(1)	$c_1(1)$	$c_2(1)$	$c_3(1)$	$c_4(1)$	
r(2)	p(2)	T(2)	$c_1(2)$	$c_2(2)$	$c_3(2)$	$c_4(2)$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
r(N)	p(N)	T(N)	$c_1(N)$	$c_2(N)$	$c_3(N)$	$c_4(N)$	

Fig. 12. Input Card File Structure for Radial Gas Data

LEGEND

All field formats are E10 or F10 except the GDTAID, GNAME1, GNAME2, GNAME3 and GNAME4 fields which are A10 and the N field which is I10.

GDTAID	Gas data identification name.
N	Number of radial points ($N \leq 201$).
R	Source radius (cm).
GNAME1 – GNAME4	Gas species identification names.
r(i)	Radial positions (cm). $0 \equiv r(1) < \ldots < r(N) \equiv R$.
T(i)	Temperature (K) at r(i).
$c_j(i)$	Concentration (mole fraction) of species j ($j=1,\ldots,4$) at r(i).

10	20	30	40	50	60	70	80
PDTAID	N	R	PNAME1	PNAME2	PNAME3		
r(1)	$T_1(1)$	$c_1(1)$	$T_2(1)$	$c_2(1)$	$T_3(1)$	$c_3(1)$	
r(2)	$T_1(2)$	$c_1(2)$	$T_2(2)$	$c_2(2)$	$T_3(2)$	$c_3(2)$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
r(N)	$T_1(N)$	$c_1(N)$	$T_2(N)$	$c_2(N)$	$T_3(N)$	$c_3(N)$	

Fig. 13. Input Card File Structure for Radial Particle Data

LEGEND

All field formats are E10 or F10 except the PDTAID and PNAME1 through PNAME3 fields which are A10, and the N field which is I10.

PDTAID	Particle data identification name.
N	Number of radial points ($N \le 201$).
R	Source radius (cm).
PNAME1 → PNAME3	Particle species identification names.
r(i)	Radial positions (cm). $0 = r(1) < \ldots < r(N) = R$.
$T_j(i)$	Temperature (K) of species j ($j=1,\ldots,3$) at r(i). Note, if $T_j(1) \le 0$, the particle temperature profile for the jth species is set equal to the gas temperature profile.
$c_j(i)$	Concentration (particles/cm^3) of species j ($j=1,\ldots,3$) at r(i).

asked for by GCOL on card type 2. The required deck structure for the data is shown in Fig. 14.

8. Particle Parameters Card. The card name is PPARAM2.* This card calls for the read in of particle scattering parameters for the particle species of interest. It is the user's responsibility to ensure that the parameters read in are consistent with the particle species asked for by PCOL on card type 2. The required deck structure for the data is defined by the FORTRAN read routine shown in Fig. 15.

9. Data Listing Card. The card name is LIST. The default output of DPREP2 is the *a priori* radial data file TAPE2. A listing of the prepared data is made if the control card LIST is included in the control card deck.

10. Listing Suppression Card. The card name is NOLIST. Listings for subsequent runs can be suppressed with the NOLIST card if listing was enabled in previous runs with a LIST card.

11. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of the results is made, and the *a priori* radial data file TAPE2 is generated.

3.3 EAPROF Data Preparation

Program EAPROF calculates transverse E/A and laser scattering efficiency functions. Its main source of input data is the *a priori* radial data file TAPE2 previously generated by DPREP2. Additional data required to run

---

* Card name PPARAM1 is reserved for future use to read in particle index of refraction data.

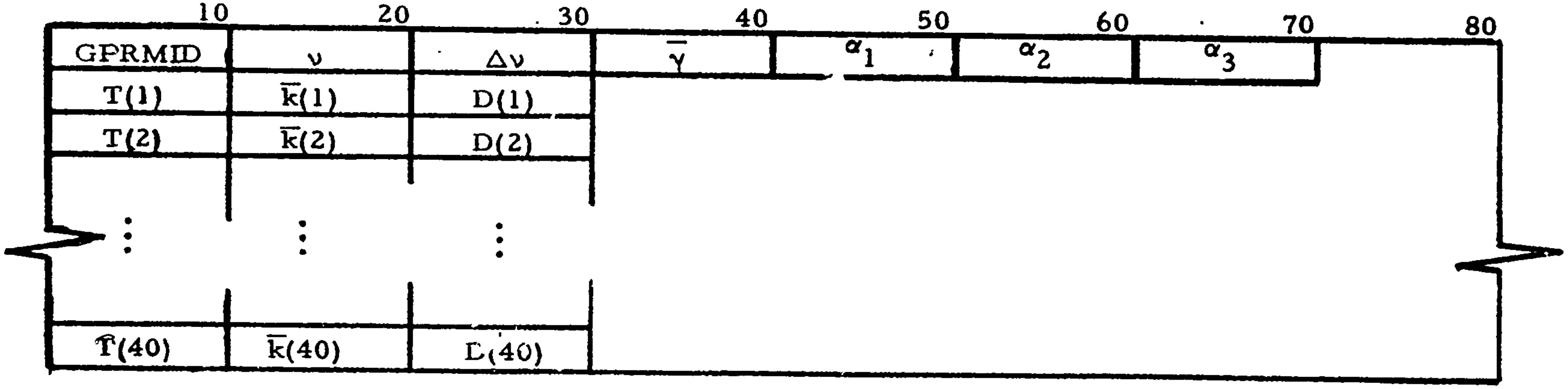

Fig. 14. Input Card File Structure for for Band Model Parameters

LEGEND

All Field formats are E10 or F10 except the GPRMID field which is A10.

GPRMID	Identification name.
ν	Spectral position (cm^{-1}).
$\Delta\nu$	Spectral resolution (cm^{-1}).
$\bar{\gamma}$	Pressure broadening coefficient (cm^{-1}/atm) for nonresonant self-broadening at STP.
α_1	Ratio of resonant self-broadening parameter to $\bar{\gamma}$ at STP.
α_2	Ratio of foreign gas broadening parameter to $\bar{\gamma}$ at STP.
α_3	Atomic weight of active gas species (amu).
T(i)	Temperature array (K). The array must be T(i) = 100i, i=1,2...,40.
$\bar{k}$(i)	Absorption coefficient for ν, $\Delta\nu$ and T(i) (cm^{-1}/atm).
D(i)	Line density parameter for ν, $\Delta\nu$ and T(i) (lines/cm^{-1}). Note, $D \equiv 1/\bar{\delta}$.

```
      READ (5, 100) PPRMID, NR, NA, WN
      READ (5, 101) (R(I), I = 1, NR)
      READ (5, 101) (SA(I), I = 1, NR)
      READ (5, 101) (SS(I), I = 1, NR)
      READ (5, 101) (ANG(J), J = 1, NA)
      DO 1 I = 1, NR
      READ (5, 101) (DS(I,J), J = 1, NA)
    1 CONTINUE
  100 FORMAT (A10, 2I10, E10.0)
  101 FORMAT (8E10.0)
```

PPRMID	Particle parameters identification name.
NR	Number of radial points (NR < 201).
NA	Number of angles (NA < 181).
WN	Wavenumber (cm^{-1}).
R(I)	Radial positions r(cm). 0 = R(1) < ... < R(NR) = R.
SA(I)	Absorption cross section σ_a (cm^2) at R(I).
SS(I)	Total scattering cross section σ_s (cm^2) at R(I).
ANG(J)	Scattering angles θ (deg). 0 = ANG(1 < ... < ANG(NA) = 180.
DS(I,J)	Differential scattering cross section $d\sigma_s/d\Omega$ (cm^2/sr) at R(I) and ANG(J).

Fig. 15. Input Routine for Particle Scattering Parameters

EAPROF are supplied by the control cards shown in Fig. 16 and described below.

1. Title Card.

2. Calculation Data Card. The card name is CALCDATA. The variable MODE (format A10) determines the gas and particle case for which calculations will be made. If MODE has the value GAS, calculations will be made as though only the gas species were optically active. The value PARTICLE implies that only the particle species is active. The value BOTH implies a coupled gas-plus-particle calculation. If MODE has the value ALL, calculations are performed for all three cases. The variables SHAPE and INHOM (each format A10) specify, respectively, the gas absorption lineshape and optical path nonuniformity approximation to be employed in the gas band model. SHAPE must be one of the values LORENTZ, DOPPLER, or VOIGT. INHOM must have either the value CG (for the Curtis-Godson approximation) or DR (for the derivation approximation). NSCAT (format I10) is the number of equal-length segments that a scattering LOS is divided into for numerical integration. Its maximum allowed value is 100. NAZ (format I10) is the number of intervals over which the 360° azimuthal angle integration is performed in integrations over solid angle. There is no dimensional limit to the value of NAZ.

3. Plume Data Card. The card name is PLMDATA. L1 (format E10) is the distance (cm) from the nozzle exit plane to the observation scanning plane. L2 (format E10) is the distance (cm) from the scanning plane to the end of the plume. TN (format E10) is the temperature (K) of the nozzle exit plane disc, and EN (format E10) is its emissivity. These data (and thus the card) are not

	10	20	30	40	50	60	70	80
1	TITLE							
2	CALCDATA	MODE	SHAPE	INHOM	NSCAT	NAZ		
3	PLMDATA	L1	L2	TN	EN			
4	RDATA							
5	RUN							

Fig. 16. EAPROF Program Control Card Formats

required if MODE = GAS on the calculation data card (card type 2).

4. Radial Data Card. The card name is RDATA. This card calls for the read in of the *a priori* radial data file. This file must be attached to the EAPROF job run as TAPE2.

5. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of the results is made, and the transverse data file TAPE3 is generated.

3.4 PARIC1 DATA PREPARATION

Program PARIC1 retrieves the radial volume extinction profile $\gamma(r)$ by Abel inversion from the transverse particle-only transmittance $\tau_p(z)$. This transmittance is read in from the transverse data file TAPE3 generated by EAPROF or DPREP1. The control cards for PARIC1 are shown in Fig. 17 and described below.

1. Title Card.

2. Transverse Data Card. The card name is ZDATA. This card calls for the read in of $\tau_p(z)$ from the transverse data file. This file must be attached to the PARIC1 job run as TAPE3.

3. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of the results is made, and the retrieved function $\gamma(r)$ is written to the retrieved radial data file TAPE4.

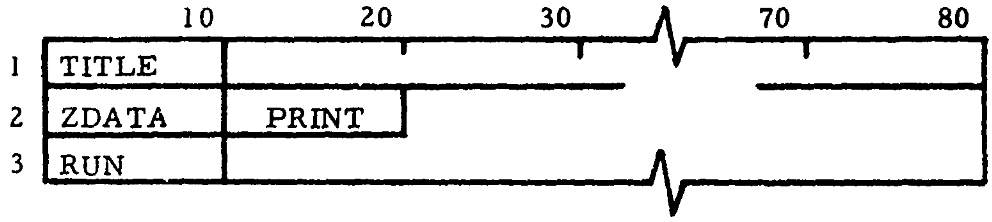

Fig. 17. PARICl Program Control Card Formats

3.5 PARIC2 Data Preparation

Program PARIC2 retrieves the radial volume scattering cross section $\beta(r)$, volume absorption cross section $\alpha(r)$, and scattering phase function $p(r,\theta)$ by iterative Abel inversion from the transverse laser scattering efficiency function $f(z,\theta)$. This function is read in from the transverse data file TAPE3 generated by EAPROF or DPREP1. The control cards for PARIC2 are shown in Fig. 18 and described below.

1. Title Card.
2. Convergence Data Card. The card name is CONVERGE. This card enters the iterative Abel inversion convergence criteria. IMAX (format I10) is the maximum number of iterations allowed (unlimited), and ERROR (format E10) is the maximum rms difference (percent) allowed between successive iterations of the retrieved scattering source function for convergence to be deemed complete.
3. Miscellaneous Data Card. The card name is NDATA. NSCAT (format I10) is the number of equal-length segments that a scattering LOS is divided into for numerical integration. Its maximum allowed value is 100. If NL (format I10) has the value 1, calculations will be performed as though the laser scattering efficiency function were defined only for a single line of sight through the plume at $z = o$. The retrieved phase function and volume scattering cross section will be constant in r. If NL $\neq$ 1, a full inversion for the radial variation of these functions is made.
4. Iteration Listing Card. The card name is LIST. Normally, the intermediate results for current phase function and transverse

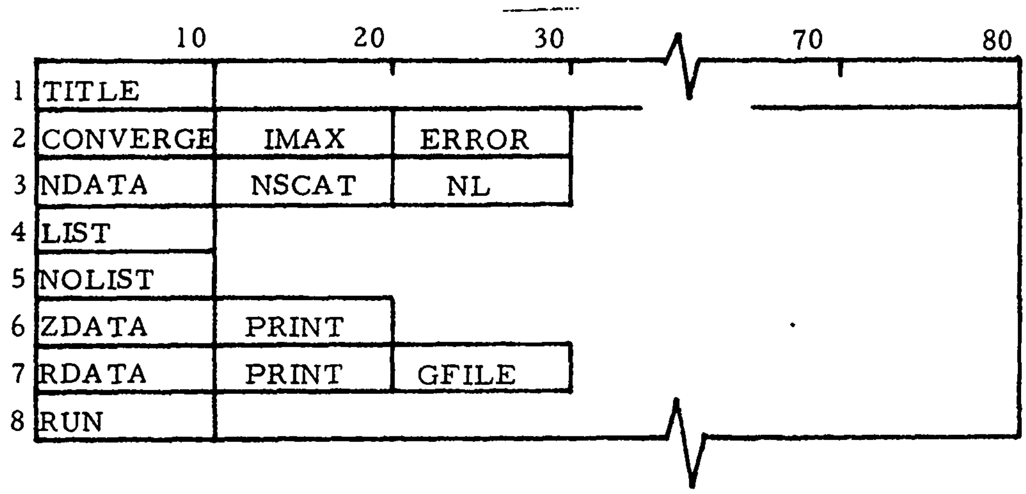

Fig. 18. PARIC2 Program Control Card Formats

profiles generated at each iteration are not listed. The LIST control card can be used to obtain them.

5. Listing Suppression Card. The card name is NOLIST. Listing of intermediate iteration results can be suppressed with the NOLIST card if listing was enabled in previous runs with a LIST card (card type 4).

6. Transverse Data Card. The card name is ZDATA. This card calls for the read in of $f(z,\theta)$ from the transverse data file. This file must be attached to the PARIC2 job run as TAPE3.

7. Radial Data Card. The card name is RDATA. This card calls for the read in of radial data. The only radial data required by this code is the volume extinction cross section $\gamma(r)$. If GFILE (format I10) has the value 2, $\gamma(r)$ is obtained from the *a priori* radial data file TAPE2. If GFILE has the value 4, $\gamma(r)$ is obtained from the retrieved radial data file TAPE4. The appropriate file must be attached to the PARIC2 job run.

8. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of the results is made, and the retrieved functions $\alpha(r)$, $\beta(r)$ and $p(r,\theta)$ are written to the retrieved radial data file TAPE4. $p(r,\theta)$ is defined on the laser scattering angle grid θ_L.

3.6 PARIC3 Data Preparation

Program PARIC3 retrieves the radial particle temperature profile $T_p(r)$ by iterative Abel inversion from the transverse particle-only radiance profile $N_p(z)$. This radiance is read in from the transverse data file TAPE3 generated by EAPROF or DPREP1. The control card formats for PARIC3 are shown in Fig. 19 and described below.

1. Title Card.

2. Convergence Data Card. The card name is CONVERGE. This card enters the iterative Abel inversion convergence criteria. IMAX (format I10) is the maximum number of iterations allowed (unlimited), and ERROR (format E10) is the maximum rms difference (percent) allowed between successive iterations of the retrieved thermal source function for convergence to be deemed complete.

3. Plume Data Card. The card name is PLMDATA. L1 (format E10) is the distance (cm) from the nozzle exit plane to the observation scanning plane. L2 (format E10) is the distance (cm) from the scanning plane to the end of the plume. TN (format E10) is the temperature (K) of the nozzle exit plane disc, and EN (format E10) is its emissivity.

4. Miscellaneous Data Card. The card name is NDATA. NSCAT (format I10) is the number of equal-length segments that a scattering line of sight is divided into for numerical integration. Its maximum allowed value is 100. NAZ (format I10) is the number of intervals over which the 360° azimuthal angle integration is performed in integrations over solid angle. There is no dimensional limit to NAZ.

5. Iteration Listing Card. The card names is LIST. Normally, the intermediate results for current thermal source function and transverse profiles generated at each iteration are not listed. The LIST control card can be used to obtain them.

	10	20	30	40	50	60	70	80
1	TITLE							
2	CONVERGE	IMAX	ERROR					
3	PLMDATA	L1	L2	TN	EN			
4	NDATA	NAZ	NSCAT					
5	LIST							
6	NOLIST							
7	AGRID	PRINT						
8	NOAGRID							
9	ZDATA	PRINT						
10	RDATA	PRINT	AFILE	BFILE	PFILE			
11	RUN							

Fig. 19. PARIC3 Program Control Card Formats

6. Listing Suppression Card. The card name is NOLIST. Listing of intermediate iteration results can be suppressed with the NOLIST card if listing was enabled in previous runs with a LIST card (card type 5).

7. Scattering Angle Integration Grid Card. (Type 1) The card name is AGRID. If this card is used, it must come before the RDATA control card (card type 10) in the control card deck. The AGRID card calls for the read in of the scattering angle grid θ that PARIC3 will use for integration over solid angle. The required deck structure for these data is shown in Fig. 11. If no AGRID card is used, the scattering grid θ for solid angle integration and the values of the scattering phase function p on that grid will be read (when the RDATA card is encountered) from the *a priori* radial data file TAPE2. If an AGRID card is used, the laser scattering grid θ_L and the values of the scattering phase function p_L on that grid will be read (when the RDATA card is encountered) from the *a priori* radial data file TAPE2 or the retrieved radial data file TAPE4, and an interpolation made to get the scattering phase function p on the grid θ read in by the AGRID card.

8. Scattering Angle Integration Grid Card (Type 2). The card name is NOAGRID. This control card restores the scattering grid condition of subsequent runs to the equivalent state of never having encountered an AGRID card.

9. Transverse Data Card. The card name is ZDATA. This card calls for the read in of $N_p(z)$ from the transverse data file. This file must be attached to the PARIC3 job run as TAPE3.

10. Radial Data Card. The card name is RDATA. This card calls for the read in of radial data. The radial data required by PARIC3 are the volume absorption and scattering cross sections, $\alpha(r)$ and $\beta(r)$, respectively, and the scattering phase function $p(r,\theta)$. If AFILE (format I10) has the value 2, the $\alpha(r)$ data is obtained from the *a priori* radial data file TAPE2. If AFILE has the value 4, $\alpha(r)$ is obtained from the retrieved radial data file TAPE4. BFILE and PFILE control, respectively, the input of $\beta(r)$ and $p(r,\theta)$ in the same way. The appropriate files must be attached to the job.

11. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of results is made, and the retrieved function $T_p(r)$ is written to the retrieved radial data file TAPE4.

3.7 GASIC Data Preparation

Program GASIC retrieves the radial gas temperature $T_g(r)$ and concentration $c_g(r)$ by coupled iterative Abel inversion from the transverse total radiance $\bar{N}(z)$ and transmittance $\bar{\tau}(z)$ profiles. These transverse functions are read in from the transverse data file TAPE3 generated by EAPROF or DPREP1. The control card formats for GASIC are shown in Fig. 20 and described below.

1. Title Card.

2. Calculation Data Card. The card name is CALCDATA. The variable MODE (format A10) determines the inversion mode. If MODE has the value GAS, an inversion is made as though the input transverse profiles $\bar{N}(z)$ and $\bar{\tau}(z)$ were for a purely gaseous plume. If MODE has the value FOOB, the first-order, off-band correction

	10	20	30	40	50	60
1	TITLE					
2	CALCDATA	MODE	SHAPE	INHOM		
3	CONVERGE	IMAX	TERROR	CERROR		
4	PLMDATA	L1	L2	TN	EN	
5	NDATA	NAZ	NSCAT			
6	LIST					
7	NOLIST					
8	GPARAM	PRINT				
9	ZDATA	PRINT				
10	RDATA	PRINT	AFILE	BFILE	PFILE	TPFILE
11	RUN					

Fig. 20. GASIC Program Control Card Formats

procedure is used for inversion, and if MODE has the value BOTH, the fully-coupled, inversion procedure is used. The variables SHAPE and INHOM (each format A10) specify, respectively, the gas absorption lineshape and optical path nonuniformity approximation to be employed in the gas band model. SHAPE must be one of the values LORENTZ, DOPPLER, or VOIGT. INHOM must have either the value CG (for the Curtis-Godson approximation) or DR (for the derivative approximation).

3. Convergence Data Card. The card name is CONVERGE. This card enters the iterative Abel inversion convergence criteria. IMAX (format I10) is the maximum number of iterations allowed (unlimited). TERROR and CERROR (each format E10) are the maximum rms differences (percent) allowed between successive iterations of the retrieved gas temperature and concentration, respectively, for convergence to be deemed complete. Both criteria must be satisfied.

4. Plume Data Card. The card name is PLMDATA. L1 (format E10) is the distance (cm) from the nozzle exit plane to the observation scanning plane. L2 (format E10) is the distance (cm) from the scanning plane to the end of the plume. TN (format E10) is the temperature (K) of the nozzle exit plane disc, and EN (format E10) is its emissivity. These data (and thus the card) are required only if MODE=BOTH on the calculation data card (card type 2).

5. Miscellaneous Data Card. The card name is NDATA. NSCAT (format I10) is the number of equal-length segments that a scattering line of sight is divided into for numerical integration. Its

maximum allowed value is 100. NAZ (format I10) is the number of intervals over which the 360° azimuthal angle integration is performed in integrations over solid angle. There is no dimensional limit to NAZ. These data (and thus the card) are required only in MODE=BOTH on the calculation data card (card type 2).

6. Iteration Listing Card. The card name is LIST. Normally, the intermediate results for current gas temperature and concentration, and transverse profiles generated at each iteration are not listed. The LIST control card can be used to obtain them.

7. Listing Suppression Card. The card name is NOLIST. Listing of intermediate iteration results can be suppressed with the NOLIST card if listing was enabled in previous runs with a LIST card (card type 6).

8. Gas Band Model Parameters Card. The card name is GPARAM. This card calls for the read in of band model parameters for the gas species of interest. The required deck structure is shown in Fig. 14.

9. Transverse Data Card. The card name is ZDATA. This card calls for the read in of transverse data from the transverse data file, which must be attached to the GASIC job run as TAPE3. The total radiance $\bar{N}(z)$, total transmittance $\bar{\tau}(z)$, particle-only radiance $N_p(z)$, and particle-only transmittance $\tau_p(z)$ functions are read in, but the latter two functions are used only if MODE=FOOB on the calculation data card (card type 2). Also, although it is not a transverse profile, but a radial one, the

total gas pressure profile $p_g(r)$ is read in at this point from the transverse data file.

10. Radial Data Card. The card name is RDATA. This card calls for the read in of radial data required in the MODE=BOTH mode of inversion. If MODE≠BOTH, the radial data (and, hence, the RDATA card) are not required. The radial data required by the MODE=BOTH inversion are the volume absorption and scattering cross sections $\alpha(r)$ and $\beta(r)$, respectively, the scattering phase function $p(r,\theta)$, and the particle temperature $T_p(r)$. If AFILE (format I10) has the value 2, the $\alpha(r)$ data is obtained from the *a priori* radial data file TAPE2. If AFILE has the value 4, $\alpha(r)$ is obtained from the retrieved radial data file TAPE4. BFILE, PFILE, and TPFILE control, respectively, the input of $\beta(r)$, $p(r,\theta)$, and $T_p(r)$ in the same way. The appropriate files must be attached to the job.

11. Execution Card. The card name is RUN. When this card is encountered, computations are begun using the data entered up to that point, an output listing of results is made, and the retrieved functions $T_g(r)$ and $c_g(r)$ are written to the retrieved radial data file TAPE4.

4. EXAMPLE RUNS

The example runs illustrated here are for the minimum smoke propellant (MSP) plume model analysis presented in Refs. 1 and 5. Example runs are given for all six of the basic codes. Synthetic data were produced with the DPREP2 and EAPROF codes, and immediate retrieval made with the PARIC1, PARIC2, PARIC3, AND GASIC codes. Upon completion of the DPREP2 run, the *a priori* radial data file TAPE2 was made permanent, and upon completion of the EAPROF run, the transverse data file TAPE3 was made permanent. These files were then attached to the inversion runs as necessary. All of the inversions used radial data from the *a priori* radial data file TAPE2, and thus the retrieved radial data file was never made permanent.

The codes were run on a CDC176 computer with SCOPE2.1 operating system, and compilation was made with the FTN compiler operating in the OPT=2 mode (slowest compilation, but fastest safe execution). For the examples given here, the code running times (compilation plus execution) are shown in Table 6. The time entered in the table for GASIC is for three runs, with MODE = GAS, FOOB, and BOTH. The full inversion mode (BOTH) accounted for nearly all of the time. The running times for the first two modes were only ~ 3 sec each.

4.1 Profile Prediction Runs

The MSP plume model contains Al_2O_3 and H_2O, respectively, as the particle and gas phases. The radial gas and particle temperature profile over the 10 cm radius of the plume is shown in Fig. 21. The gas pressure, gas concentration, and particle loading profiles are constant in radius with values p = 1 atm, c_g = 0.15, and n_p = $10^5/cm^3$. The H_2O band model parameters used (Ref.

Table 6. Code Running Times.

Code	Time (sec)
DPREP2	<1
EAPROF	84
PARIC1	<1
PARIC2	2
PARIC3	19
GASIC	590*

*Total for three inversion runs.

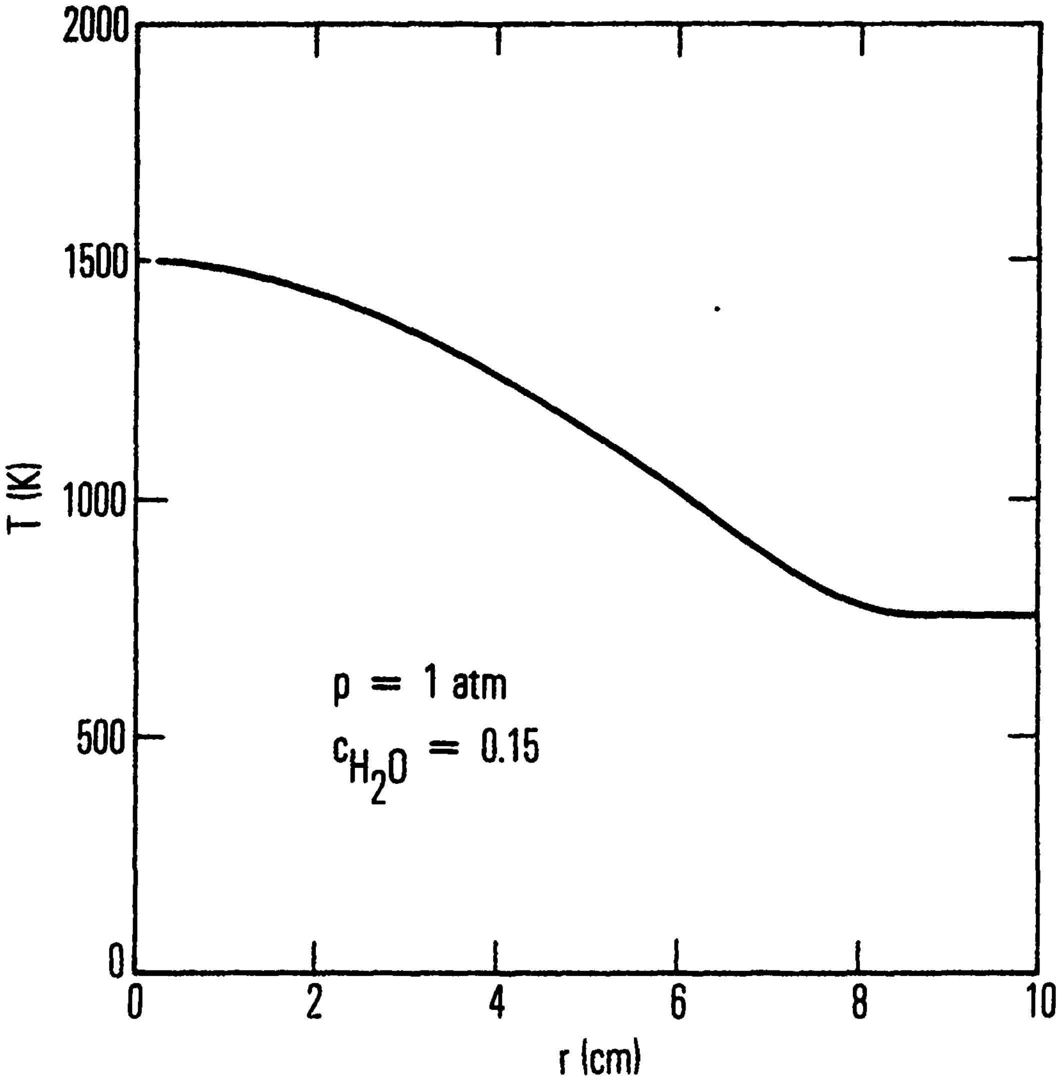

Fig. 21. MSP Gas and Particle Temperature Profile

1) are shown in Fig. 22. The nonresonant, self-broadening parameter is $\gamma_o = 0.07394\ cm^{-1}/atm$. The efficiency of resonant self-broadening is 6.53, and the efficiency for foreign gas broadening is 1.00. Particle scattering cross sections (Ref. 1) were computed with Mie theory, the particle size distribution shown in Fig. 23, and index of refraction m = 2.0 - 0.01i. The result for the absorption and total scattering cross sections are $\sigma_a = 3.20 \times 10^{-9}\ cm^2$ and $\sigma_s = 5.58 \times 10^{-8}\ cm^2$. The differential scattering cross section is shown in Fig. 24 (the result for $\kappa = 0.01$ is used here). These scattering parameters were assumed to be constants in radius. These data were prepared on the 11-point scattering grid θ = 0, 5, 15, 25, 35, 45, 60, 90, 120, 150, and 180°. The manner in which this grid covers the scattering integral integrand is shown in Fig. 25. The laser scattering efficiency function was calculated on the 8-point grid θ_L = 0, 10, 20, 30, 45, 90, 135, and 180°. Radial and transverse calculations were performed by dividing the plume into 10 equal-size zones.

The input data for DPREP2 that reflects all of these conditions is listed in Fig. 26. The output of DPREP2 is listed in Fig. 27. These results (as saved on TAPE2) are the principle input for EAPROF. Listings of the actual input date to DPREP2 have been suppressed by deleting the PRINT variable on the control cards.

Additional input data for EAPROF are listed in Fig. 28. Calculations were performed for gas-only, particle-only, and coupled gas-plus-particle conditions. The Lorentz lineshape and Curtis-Godson nonuniformity approximation were used in the gas band model. Scattering lines of sight were divided into 10 segments for numerical integration. Azimuthal angle integration was performed with a 16-segment grid. The distance from the exit plane to the observation plane was taken as 3 cm, and the distance from the observation

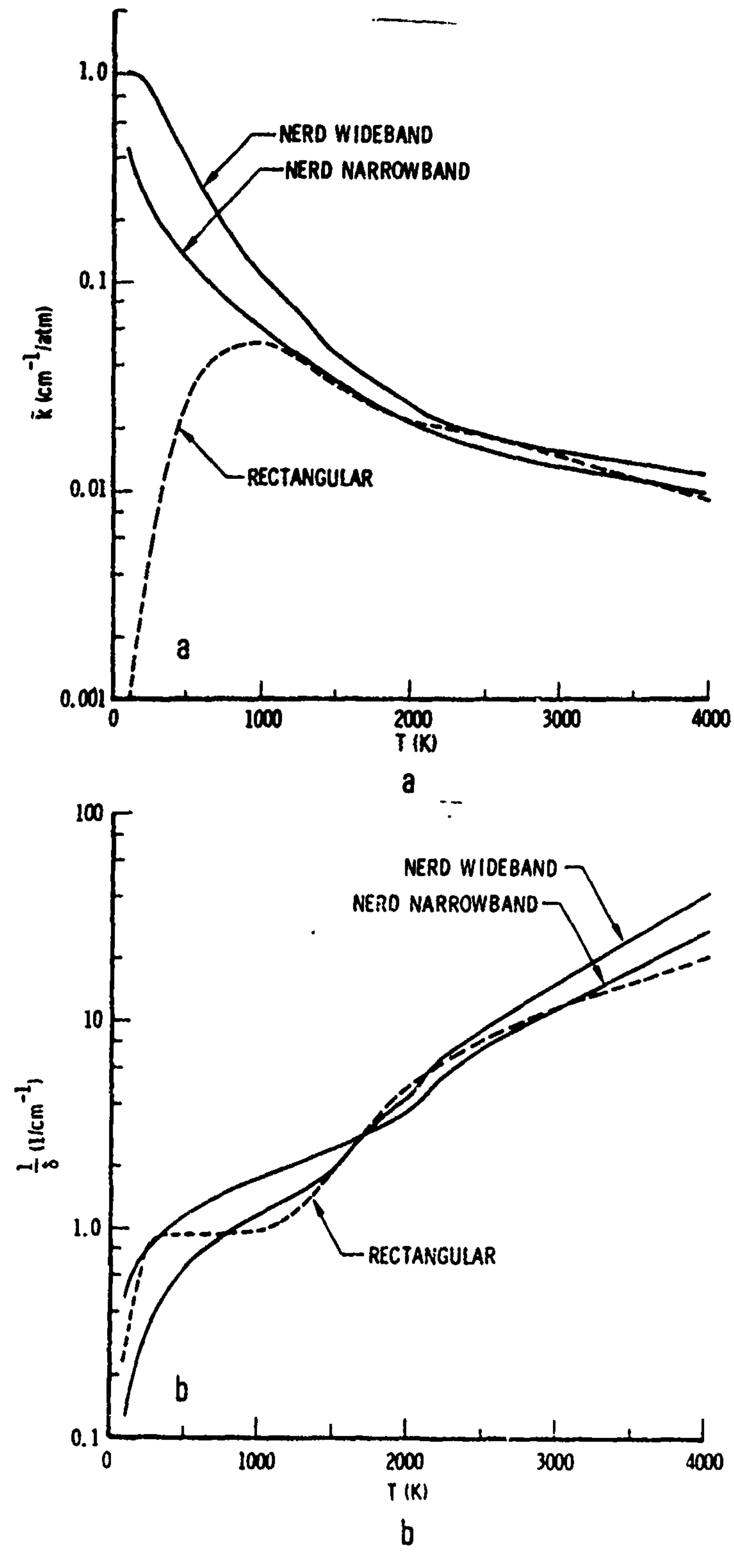

Fig. 22. H_2O Band Model Parameters

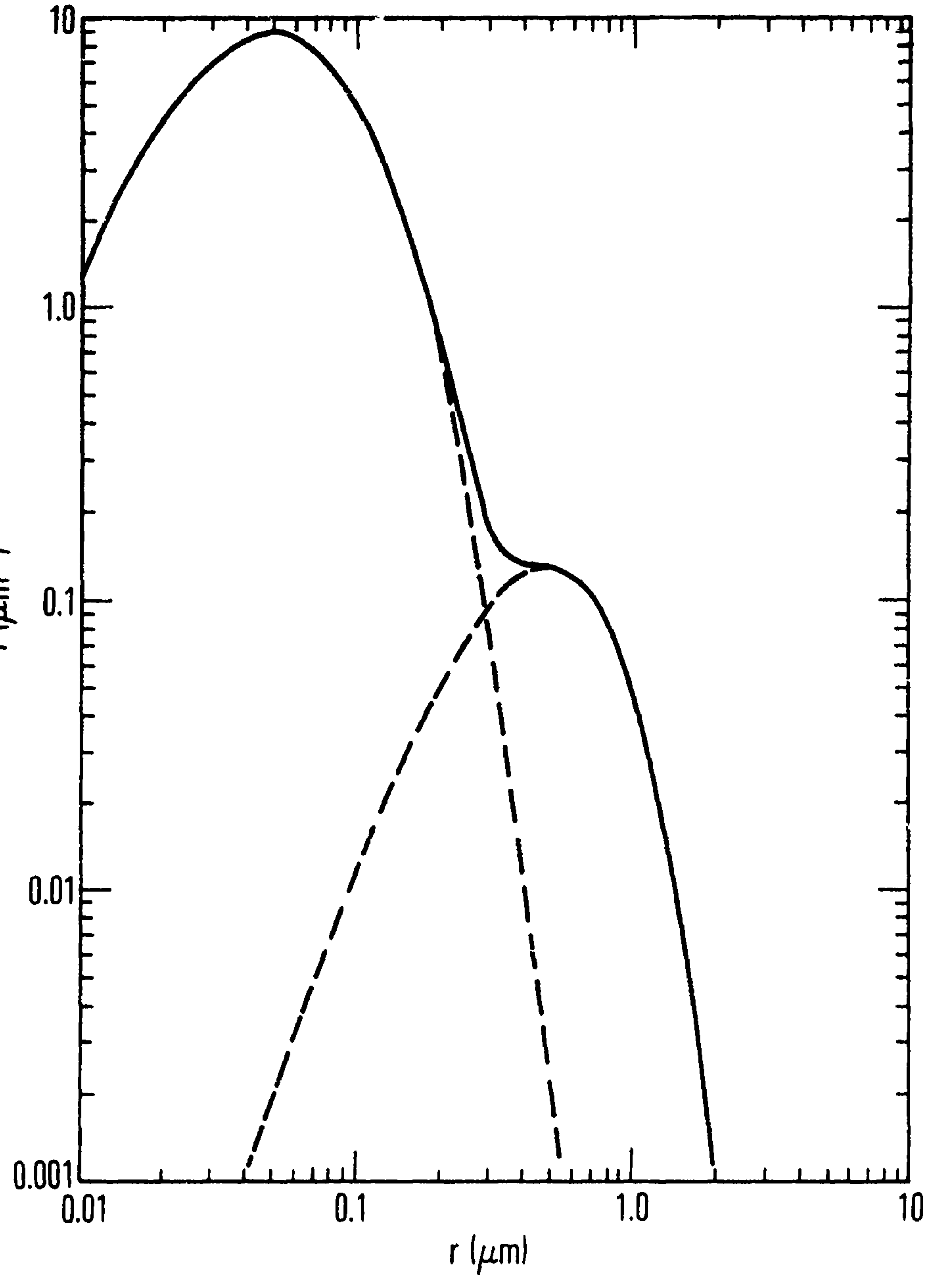

Fig. 23. Al_2O_3 Size Distribution

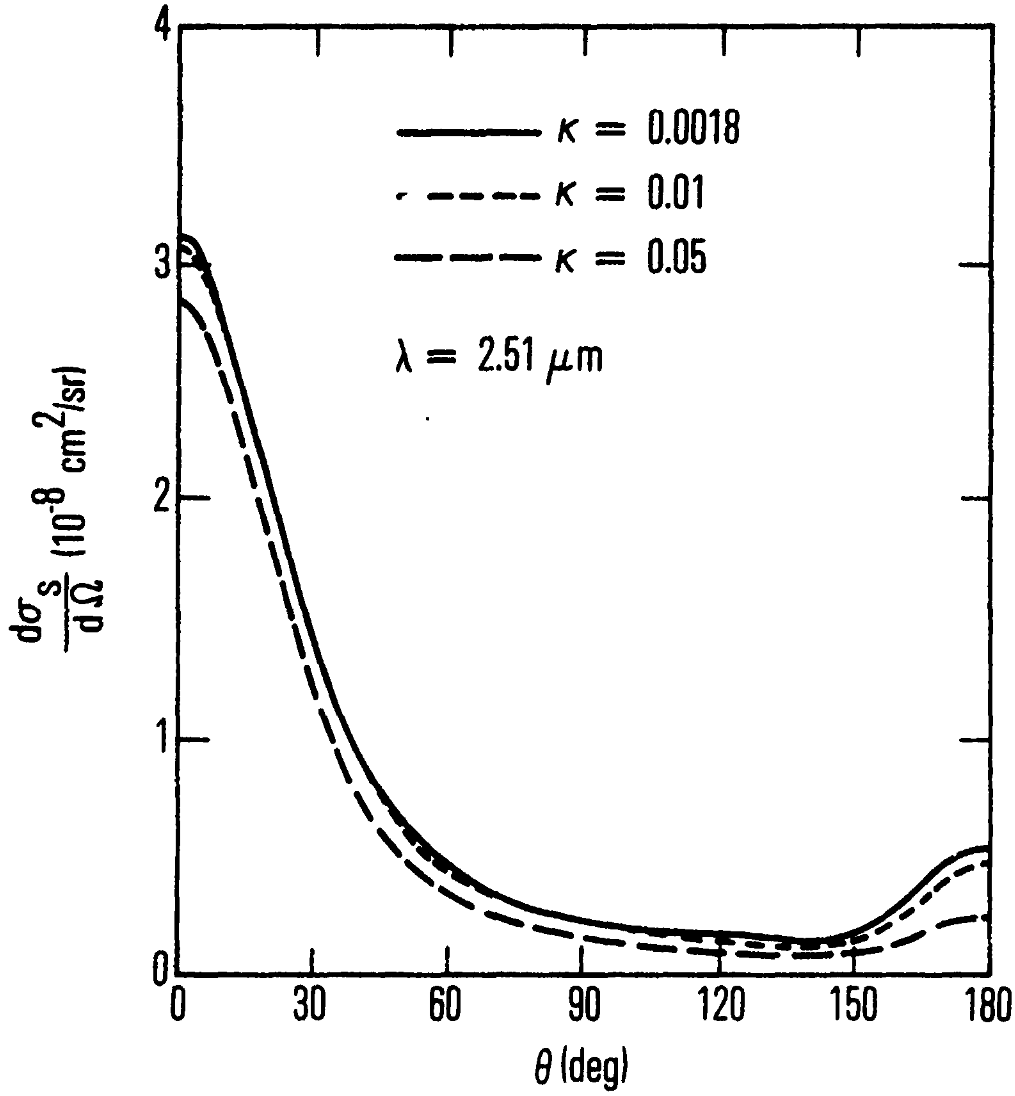

Fig. 24. Differential Scattering Cross Section for Al_2O_3

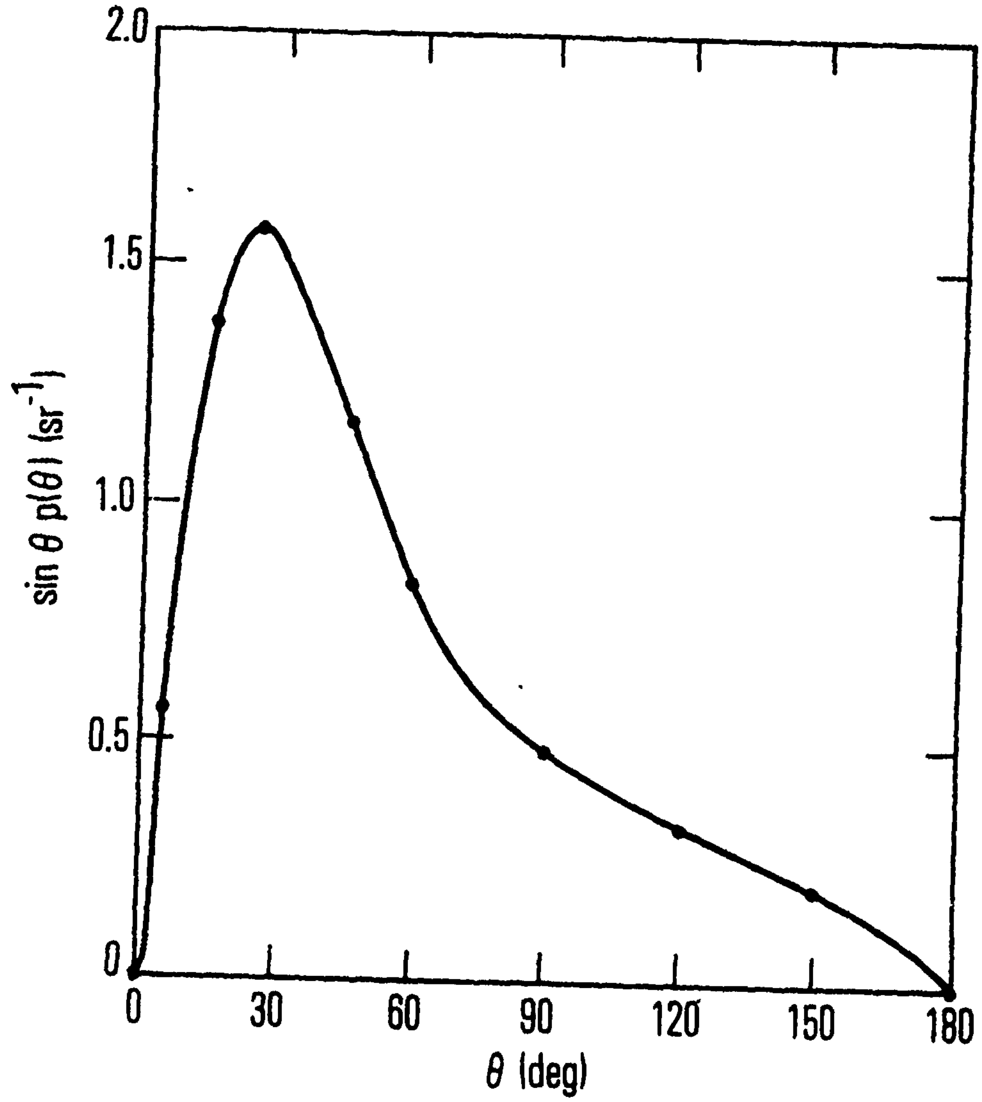

Fig. 25. Coverage of the Scattering Integral Weighting Function by the 11-Point Scattering Angle Grid

```
TITLE     MSP MODEL ANALYSIS -- AL203/H20
DATA            19          1           1
AGRID
 SCATGRID1      11          0.          5.         15.         25.         35.         45.
     60.        90.       120.        150.        180.
LGRID
LASERGRID1       8          0.         10.         20.         30.         45.         90.
    135.       180.
GDATA
     MSP-G      11         10.         H20         CO2          CO
        0.      1.0      1500.        0.15        0.15        0.35
        1.      1.0      1475.        0.15        0.15        0.35
        2.      1.0      1435.        0.15        0.15        0.35
        3.      1.0      1365.        0.15        0.15        0.35
        4.      1.0      1280.        0.15        0.15        0.35
        5.      1.0      1165.        0.15        0.15        0.35
        6.      1.0      1035.        0.15        0.15        0.35
        7.      1.0       888.        0.15        0.15        0.35
        8.      1.0       765.        0.15        0.15        0.35
        9.      1.0       750.        0.15        0.15        0.35
       10.      1.0       750.        0.15        0.15        0.35
PDATA
     MSP-P       2         10.       AL203
        0.      -1.       1.E5
       10.      -1.       1.E5
GPARAM
  MEROH20W    3985.0      300.     0.07394        6.53        1.00        18.
      100.  .4262E+00  .1242E+00
      200.  .2686E+00  .2722E+00
      300.  .1972E+00  .4058E+00
      400.  .1614E+00  .5306E+00
      500.  .1256E+00  .6658E+00
      600.  .1077E+00  .7770E+00
      700.  .8955E-01  .8926E+00
      800.  .7672E-01  .9990E+00
      900.  .6836E-01  .1096E+01
     1000.  .6000E-01  .1189E+01
     1100.  .5460E-01  .1318E+01
     1200.  .4920E-01  .1452E+01
     1300.  .4380E-01  .1592E+01
     1400.  .3839E-01  .1738E+01
     1500.  .3299E-01  .1893E+01
     1600.  .3057E-01  .2359E+01
     1700.  .2815E-01  .2828E+01
     1800.  .2573E-01  .3313E+01
     1900.  .2331E-01  .3822E+01
     2000.  .2089E-01  .4363E+01
     2100.  .1980E-01  .5327E+01
     2200.  .1871E-01  .6312E+01
     2300.  .1762E-01  .7323E+01
     2400.  .1652E-01  .8362E+01
     2500.  .1543E-01  .9435E+01
     2600.  .1499E-01  .1057E+02
     2700.  .1455E-01  .1169E+02
     2800.  .1411E-01  .1283E+02
     2900.  .1367E-01  .1397E+02
```

Fig. 26. DPREP2 Input Data Listing (Sheet 1 of 2)

```
  3000.  .1323E-01  .1511E+02
  3100.  .1254E-01  .1665E+02
  3200.  .1246E-01  .1836E+02
  3300.  .1209E-01  .2027E+02
  3400.  .1174E-01  .2241E+02
  3500.  .1140E-01  .2479E+02
  3600.  .1107E-01  .2745E+02
  3700.  .1075E-01  .3043E+02
  3800.  .1044E-01  .3378E+02
  3900.  .1015E-01  .3753E+02
  4000.  .9861E-02  .4175E+02
PPARAM2
AO/B/2.5/2            2         21      3985.
        0.          18.
  3.20E-9      3.20E-9
  5.58E-8      5.58E-8
        0.           5.        10.        15.        20.        30.        40.        50.
       60.          70.        80.        90.       100.       110.       120.       130.
      140.         150.       160.       170.       180.
  3.06E-8      2.98E-8    2.75E-8    2.42E-8    2.06E-8    1.39E-8    9.14E-9    6.18E-9
  4.39E-9      3.29E-9    2.59E-9    2.15E-9    1.87E-9    1.71E-9    1.62E-9    1.53E-9
  1.47E-9      1.70E-9    2.62E-9    4.06E-9    4.82E-9
  3.06E-8      2.98E-8    2.75E-8    2.42E-8    2.06E-8    1.39E-8    9.14E-9    6.18E-9
  4.39E-9      3.29E-9    2.59E-9    2.15E-9    1.87E-9    1.71E-9    1.62E-9    1.53E-9
  1.47E-9      1.70E-9    2.62E-9    4.06E-9    4.82E-9
LIST
RUN
```

Fig. 26. DPREP2 Input Data Listing (Sheet 2 of 2)

```
***************************************PROGRAM DPREP2 OUTPUT****************************************

JOB TITLE                                  MSP MODEL ANALYSIS -- AL203/H20
GAS SPECIES                                       H20
PARTICLE SPECIES                                AL203
GAS WAVENUMBER(CM-1)                        3.985E+03
PARTICLE WAVENUMBER(CM-1)                   3.985E+03
PLUME RADIUS(CM)                            1.000E+01
NUMBER OF ZONES                                    10
NUMBER OF SCATTERING ANGLES                        11
NUMBER OF LASER ANGLES                              8

SCATTERING ANGLE GRID IDNAME                SCATGRID1
LASER ANGLE GRID IDNAME                    LASERGRID1
GAS DATA IDNAME                                 MSP-G
PARTICLE DATA IDNAME                            MSP-P
GAS BAND MODEL PARAMETERS IDNAME             NERDH20W
PARTICLE SCATTERING DATA IDNAME            A0/B/2.5/2
REFRACTIVE INDEX IDNAME
```

Fig. 27. DPREP2 Standard Output Listing (Sheet 1 of 11)

RADIAL GAS/PARTICLE DATA

INDEX	R(CM)	P(ATM)	TG(DEGK)	TP(DEGK)	CG(MF)	CP(1/CM3)
1	0.	1.000E+00	1.500E+03	1.500E+03	1.500E-01	1.000E+05
2	1.000E+00	1.000E+00	1.475E+03	1.475E+03	1.500E-01	1.000E+05
3	2.000E+00	1.000E+00	1.435E+03	1.435E+03	1.500E-01	1.000E+05
4	3.000E+00	1.000E+00	1.365E+03	1.365E+03	1.500E-01	1.000E+05
5	4.000E+00	1.000E+00	1.280E+03	1.280E+03	1.500E-01	1.000E+05
6	5.000E+00	1.000E+00	1.165E+03	1.165E+03	1.500E-01	1.000E+05
7	6.000E+00	1.000E+00	1.035E+03	1.035E+03	1.500E-01	1.000E+05
8	7.000E+00	1.000E+00	8.800E+02	8.800E+02	1.500E-01	1.000E+05
9	8.000E+00	1.000E+00	7.650E+02	7.650E+02	1.500E-01	1.000E+05
10	9.000E+00	1.000E+00	7.500E+02	7.500E+02	1.500E-01	1.000E+05
11	1.000E+01	1.000E+00	7.500E+02	7.500E+02	1.500E-01	1.000E+05

Fig. 27. DPREP2 Standard Output Listing (Sheet 2 of 11)

RADIAL BAND MODEL PARAMETERS

INDEX	R(CM)	K(CM-1/ATM)	D(1/CM-1)	WL(CM-1)	WD(CM-1)
1	0.	3.299E-02	1.893E+00	4.473E-02	1.298E-02
2	1.000E+00	3.434E-02	1.853E+00	4.521E-02	1.287E-02
3	2.000E+00	3.650E-02	1.791E+00	4.603E-02	1.270E-02
4	3.000E+00	4.026E-02	1.686E+00	4.755E-02	1.238E-02
5	4.000E+00	4.488E-02	1.564E+00	4.959E-02	1.199E-02
6	5.000E+00	5.109E-02	1.405E+00	5.276E-02	1.144E-02
7	6.000E+00	5.777E-02	1.230E+00	5.708E-02	1.078E-02
8	7.000E+00	6.965E-02	1.077E+00	6.365E-02	9.942E-03
9	8.000E+00	8.078E-02	9.628E-01	7.002E-02	9.270E-03
10	9.000E+00	8.270E-02	9.470E-01	7.097E-02	9.178E-03
11	1.000E+01	8.270E-02	9.470E-01	7.097E-02	9.178E-03

Fig. 27. DPREP2 Standard Output Listing (Sheet 3 of 11)

RADIAL PARTICLE PARAMETERS

INDEX	R(CM)	N	K	SIGA(CM2)	SIGS(CM2)	A(CM-1)	B(CM-1)
1	0.	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
2	1.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
3	2.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
4	3.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
5	4.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
6	5.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
7	6.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
8	7.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
9	8.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
10	9.000E+00	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03
11	1.000E+01	0.	0.	3.200E-09	5.580E-08	3.200E-04	5.580E-03

Fig. 27. DPREP2 Standard Output Listing (Sheet 4 of 11)

```
SCATTERING PARAMETERS ON ANGLE INTEGRATION GRID

RINDEX=     1
INDEX      ANG(DEG)        DSS(CM2/SR)       P(SR-1)

  1        0.               2.933E-08        6.606E+00
  2        5.000E+00        2.856E-08        6.433E+00
  3        1.500E+01        2.320E-08        5.224E+00
  4        2.500E+01        1.646E-08        3.706E+00
  5        3.500E+01        1.081E-08        2.434E+00
  6        4.500E+01        7.127E-09        1.605E+00
  7        6.000E+01        4.208E-09        9.477E-01
  8        9.000E+01        2.061E-09        4.641E-01
  9        1.200E+02        1.553E-09        3.497E-01
 10        1.500E+02        1.630E-09        3.670E-01
 11        1.800E+02        4.620E-09        1.040E+00
RINDEX=     2
INDEX      ANG(DEG)        DSS(CM2/SR)       P(SR-1)

  1        0.               2.933E-08        6.606E+00
  2        5.000E+00        2.856E-08        6.433E+00
  3        1.500E+01        2.320E-08        5.224E+00
  4        2.500E+01        1.646E-08        3.706E+00
  5        3.500E+01        1.081E-08        2.434E+00
  6        4.500E+01        7.127E-09        1.605E+00
  7        6.000E+01        4.208E-09        9.477E-01
  8        9.000E+01        2.061E-09        4.641E-01
  9        1.200E+02        1.553E-09        3.497E-01
 10        1.500E+02        1.630E-09        3.670E-01
 11        1.800E+02        4.620E-09        1.040E+00
RINDEX=     3
INDEX      ANG(DEG)        DSS(CM2/SR)       P(SR-1)

  1        0.               2.933E-08        6.606E+00
  2        5.000E+00        2.856E-08        6.433E+00
  3        1.500E+01        2.320E-08        5.224E+00
  4        2.500E+01        1.646E-08        3.706E+00
  5        3.500E+01        1.081E-08        2.434E+00
  6        4.500E+01        7.127E-09        1.605E+00
  7        6.000E+01        4.208E-09        9.477E-01
  8        9.000E+01        2.061E-09        4.641E-01
  9        1.200E+02        1.553E-09        3.497E-01
 10        1.500E+02        1.630E-09        3.670E-01
 11        1.800E+02        4.620E-09        1.040E+00
RINDEX=     4
INDEX      ANG(DEG)        DSS(CM2/SR)       P(SR-1)

  1        0.               2.933E-08        6.606E+00
```

Fig. 27. DPREP2 Standard Output Listing (Sheet 5 of

2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 5

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 6

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 7

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

Fig. 27. DPREP2 Standard Output Listing (Sheet 6 of 11)

RINDEX= 8

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 9

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 10

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01
8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

RINDEX= 11

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.933E-08	6.606E+00
2	5.000E+00	2.856E-08	6.433E+00
3	1.500E+01	2.320E-08	5.224E+00
4	2.500E+01	1.646E-08	3.706E+00
5	3.500E+01	1.081E-08	2.434E+00
6	4.500E+01	7.127E-09	1.605E+00
7	6.000E+01	4.208E-09	9.477E-01

Fig. 27. DPREP2 Standard Output Listing (Sheet 7 of 1

8	9.000E+01	2.061E-09	4.641E-01
9	1.200E+02	1.553E-09	3.497E-01
10	1.500E+02	1.630E-09	3.670E-01
11	1.800E+02	4.620E-09	1.040E+00

Fig. 27. DPREP2 Standard Output Listing (Sheet 8 of 11)

SCATTERING PARAMETERS ON LASER ANGLE GRID

RINDEX= 1

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 2

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 3

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 4

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 5

Fig. 27. DPREP2 Standard Output Listing (Sheet 9 of 11)

```
INDEX     ANG(DEG)      DSS(CM2/SR)     P(SR-1)

  1       0.            2.702E-08       6.084E+00
  2       1.000E+01     2.428E-08       5.468E+00
  3       2.000E+01     1.819E-08       4.096E+00
  4       3.000E+01     1.227E-08       2.764E+00
  5       4.500E+01     6.564E-09       1.478E+00
  6       9.000E+01     1.898E-09       4.275E-01
  7       1.350E+02     1.321E-09       2.975E-01
  8       1.800E+02     4.256E-09       9.584E-01

RINDEX=     6

INDEX     ANG(DEG)      DSS(CM2/SR)     P(SR-1)

  1       0.            2.702E-08       6.084E+00
  2       1.000E+01     2.428E-08       5.468E+00
  3       2.000E+01     1.819E-08       4.096E+00
  4       3.000E+01     1.227E-08       2.764E+00
  5       4.500E+01     6.564E-09       1.478E+00
  6       9.000E+01     1.898E-09       4.275E-01
  7       1.350E+02     1.321E-09       2.975E-01
  8       1.800E+02     4.256E-09       9.584E-01

RINDEX=     7

INDEX     ANG(DEG)      DSS(CM2/SR)     P(SR-1)

  1       0.            2.702E-08       6.084E+00
  2       1.000E+01     2.428E-08       5.468E+00
  3       2.000E+01     1.819E-08       4.096E+00
  4       3.000E+01     1.227E-08       2.764E+00
  5       4.500E+01     6.564E-09       1.478E+00
  6       9.000E+01     1.898E-09       4.275E-01
  7       1.350E+02     1.321E-09       2.975E-01
  8       1.800E+02     4.256E-09       9.584E-01

RINDEX=     8

INDEX     ANG(DEG)      DSS(CM2/SR)     P(SR-1)

  1       0.            2.702E-08       6.084E+00
  2       1.000E+01     2.428E-08       5.468E+00
  3       2.000E+01     1.819E-08       4.096E+00
  4       3.000E+01     1.227E-08       2.764E+00
  5       4.500E+01     6.564E-09       1.478E+00
  6       9.000E+01     1.898E-09       4.275E-01
  7       1.350E+02     1.321E-09       2.975E-01
  8       1.800E+02     4.256E-09       9.584E-01

RINDEX=     9

INDEX     ANG(DEG)      DSS(CM2/SR)     P(SR-1)

  1       0.            2.702E-08       6.084E+00
  2       1.000E+01     2.428E-08       5.468E+00
  3       2.000E+01     1.819E-08       4.096E+00
  4       3.000E+01     1.227E-08       2.764E+00
```

Fig. 27. DPREP2 Standard Output Listing (Sheet 10 of 11)

5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 10

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

RINDEX= 11

INDEX	ANG(DEG)	DSS(CM2/SR)	P(SR-1)
1	0.	2.702E-08	6.084E+00
2	1.000E+01	2.428E-08	5.468E+00
3	2.000E+01	1.819E-08	4.096E+00
4	3.000E+01	1.227E-08	2.764E+00
5	4.500E+01	6.564E-09	1.478E+00
6	9.000E+01	1.898E-09	4.275E-01
7	1.350E+02	1.321E-09	2.975E-01
8	1.800E+02	4.256E-09	9.584E-01

Fig. 27. DPREP2 Standard Output Listing (Sheet 11 of 11)

```
TITLE       MSP MODEL ANALYSIS -- AL2O3/H2O
CALCDATA           ALL    LORENTZ        CG        10        16
PLMDATA             3.         57.      800.      0.75
RDATA
RUN
```

Fig. 28. EAPROF Input Data Listing

plane to the end of the plume was fixed at 57 cm. The exit plane was modeled as a flat disc with uniform temperature T = 800 K and emissivity ε = 0.75.

The output of EAPROF is listed in Fig. 29. These results (as saved on TAPE3) are the synthesized input transverse data for the inversion codes.

4.2 Retrieval Runs

The input data for the PARIC1, PARIC2, PARIC3, and GASIC runs are listed in Figs. 30, 32, 34, and 36, respectively. For all runs, calculation condition (e.g., number of zones, plume size, lineshape) required a repeat of the conditions used in the prediction run using DPREP2 and EAPROF. Any radial data required was always obtained from the *a priori* radial data file TAPE2 rather than from the retrieved radial data file TAPE4. Listing of input radial and transverse data was suppressed. For PARIC2, PARIC3, and GASIC, a convergence criterion of 0.1% within 30 iterations was set. For GASIC, this criterion applied to both temperature and concentration. Also for GASIC, a multiple run was made for each of the three retrieval modes. The outputs of the four runs are listed in Figs. 31, 33, 35, and 37.

```
*********************************SUMMARY LISTING OF INPUT DATA*********************************
INPUT(TAPE2) TITLE                          MSP MODEL ANALYSIS -- AL203/H20
OUTPUT(TAPE3) TITLE                         MSP MODEL ANALYSIS -- AL203/H20
PLUME RADIUS(CM)                             1.000E+01
LINESHAPE                                      LORENTZ
INHOMOGENEITY APPROXIMATION                         CG
NUMBER OF ZONES                                     10
GAS SPECIES                                        H20
BAND MODEL PARAMETER IDNAME                   NERDH20W
GAS DATA IDNAME                                  MSP-G
PARTICLE SPECIES                                 AL203
SCATTERING PARAMETER IDNAME                 A0/B/2.5/2
PARTICLE DATA IDNAME                             MSP-P
SCATTERING GRID IDNAME                      SCATGRID1
NUMBER OF SCATTERING ANGLES                         11
NUMBER OF AZIMUTHAL ANGLES                          16
NUMBER OF SIGMA-AXIS INTERVALS                      10
DISTANCE TO EXIT PLANE(CM)                   3.000E+00
DISTANCE TO END PLANE(CM)                    5.700E+01
NOZZLE TEMPERATURE(DEGK)                     8.000E+02
NOZZLE EMISSIVITY                            7.500E-01
GAS/PARTICLE MODE                                  ALL
LASER ANGLE ARRAY IDNAME                    LASERGRID1
NUMBER OF LASER SCAT ANGLES                          8
```

Fig. 29. EAPROF Standard Output Listing (Sheet 1 of 13)

*** E/A PROFILE RESULTS ***

J	********** GAS-ONLY RADIANCE(W/CM2-SR -CM-1)	RADIANCE(W/CM2-SR -MICRON)	********** ABS	TRAN	******** PARTICLE-ONLY RADIANCE(W/CM2-SR -CM-1)	RADIANCE(W/CM2-SR -MICRON)	EXT	******** TRAN	********* GAS/PARTICLE RADIANCE(W/CM2-SR -CM-1)	RADIANCE(W/CM2-SR -MICRON)	EXT	******** TRAN
1	5.526E-05	8.776E-02	.1116	.8884	5.565E-06	8.837E-03	.1113	.8887	5.961E-05	9.466E-02	.2105	.7895
2	5.365E-05	8.520E-02	.1118	.8882	5.355E-06	8.504E-03	.1108	.8892	5.786E-05	9.188E-02	.2102	.7898
3	4.916E-05	7.807E-02	.1124	.8876	4.820E-06	7.655E-03	.1092	.8908	5.299E-05	8.415E-02	.2093	.7907
4	4.189E-05	6.652E-02	.1132	.8868	4.015E-06	6.376E-03	.1065	.8935	4.518E-05	7.175E-02	.2076	.7924
5	3.262E-05	5.180E-02	.1137	.8863	3.139E-06	4.985E-03	.1025	.8975	3.534E-05	5.612E-02	.2045	.7955
6	2.232E-05	3.544E-02	.1136	.8864	2.264E-06	3.595E-03	.0971	.9029	2.447E-05	3.886E-02	.1997	.8003
7	1.312E-05	2.083E-02	.1125	.8875	1.551E-06	2.463E-03	.0901	.9099	1.478E-05	2.347E-02	.1924	.8076
8	6.733E-06	1.069E-02	.1096	.8904	9.759E-07	1.550E-03	.0808	.9192	7.920E-06	1.258E-02	.1815	.8185
9	3.879E-06	6.160E-03	.1003	.8997	6.713E-07	1.066E-03	.0684	.9316	4.735E-06	7.520E-03	.1618	.8382
10	2.870E-06	4.558E-03	.0794	.9206	4.170E-07	6.622E-04	.0501	.9499	3.409E-06	5.413E-03	.1255	.8745
11	0.	0.	0.0000	1.0000	0.	0.	0.0000	1.0000	0.	0.	0.0000	1.0000

Fig. 29. EAPROF Standard Output Listing (Sheet 2 of 13)

```
*************************************LASER SCATTERING FUNCTIONS*************************************

TRANSVERSE INDEX =  1

    INDEX   ANGLE(DEG)       F

      1        0.000      4.802E-02
      2       10.000      4.317E-02
      3       20.000      3.237E-02
      4       30.000      2.187E-02
      5       45.000      1.173E-02
      6       90.000      3.421E-03
      7      135.000      2.365E-03
      8      180.000      7.581E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 3 of 13)

```
************************************LASER SCATTERING FUNCTIONS************************************

TRANSVERSE INDEX =  2

    INDEX  ANGLE(DEG)       F

      1        0.000    4.781E-02
      2       10.000    4.293E-02
      3       20.000    3.216E-02
      4       30.000    2.171E-02
      5       45.000    1.163E-02
      6       90.000    3.383E-03
      7      135.000    2.344E-03
      8      180.000    7.548E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 4 of 13)

```
******************************************LASER SCATTERING FUNCTIONS******************************************

TRANSVERSE INDEX =   3

    INDEX   ANGLE(DEG)        F

      1        0.000      4.716E-02
      2       10.000      4.231E-02
      3       20.000      3.165E-02
      4       30.000      2.135E-02
      5       45.000      1.142E-02
      6       90.000      3.313E-03
      7      135.000      2.302E-03
      8      180.000      7.445E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 5 of 13)

```
************************************LASER SCATTERING FUNCTIONS************************************

TRANSVERSE INDEX =   4

    INDEX   ANGLE(DEG)         F

      1        0.000       4.606E-02
      2       10.000       4.127E-02
      3       20.000       3.085E-02
      4       30.000       2.078E-02
      5       45.000       1.110E-02
      6       90.000       3.208E-03
      7      135.000       2.237E-03
      8      180.000       7.270E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 6 of 13)

```
*********************************LASER SCATTERING FUNCTIONS*************************************

TRANSVERSE INDEX =  5

    INDEX  ANGLE(DEG)       F

      1       0.000     4.445E-02
      2      10.000     3.979E-02
      3      20.000     2.970E-02
      4      30.000     1.998E-02
      5      45.000     1.065E-02
      6      90.000     3.067E-03
      7     135.000     2.148E-03
      8     180.000     7.015E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 7 of 13)

```
*****************************************LASER SCATTERING FUNCTIONS*****************************************

TRANSVERSE INDEX =  6

    INDEX  ANGLE(DEG)      F

      1       0.000     4.225E-02
      2      10.000     3.778E-02
      3      20.000     2.817E-02
      4      30.000     1.892E-02
      5      45.000     1.007E-02
      6      90.000     2.886E-03
      7     135.000     2.030E-03
      8     180.000     6.666E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 8 of 13)

************************************LASER SCATTERING FUNCTIONS************************************

TRANSVERSE INDEX = 7

INDEX	ANGLE(DEG)	F
1	0.000	3.933E-02
2	10.000	3.513E-02
3	20.000	2.616E-02
4	30.000	1.755E-02
5	45.000	9.312E-03
6	90.000	2.656E-03
7	135.000	1.877E-03
8	180.000	6.205E-03

Fig. 29. EAPROF Standard Output Listing (Sheet 9 of 13)

```
*****************************************LASER SCATTERING FUNCTIONS**************************************

TRANSVERSE INDEX =  3

    INDEX   ANGLE(DEG)        F

      1        0.000       3.547E-02
      2       10.000       3.164E-02
      3       20.000       2.353E-02
      4       30.000       1.576E-02
      5       45.000       8.338E-03
      6       90.000       2.365E-03
      7      135.000       1.680E-03
      8      180.000       5.594E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 10 of 13)

```
*********************************LASER SCATTERING FUNCTIONS*********************************

TRANSVERSE INDEX =   9

    INDEX   ANGLE(DEG)         F

      1        0.000       3.020E-02
      2       10.000       2.691E-02
      3       20.000       1.997E-02
      4       30.000       1.335E-02
      5       45.000       7.035E-03
      6       90.000       1.985E-03
      7      135.000       1.417E-03
      8      180.000       4.762E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 11 of 13)

```
************************************LASER SCATTERING FUNCTIONS************************************

TRANSVERSE INDEX = 10

    INDEX   ANGLE(DEG)        F

      1        0.000      2.237E-02
      2       10.000      1.990E-02
      3       20.000      1.474E-02
      4       30.000      9.820E-03
      5       45.000      5.147E-03
      6       90.000      1.445E-03
      7      135.000      1.036E-03
      8      180.000      3.526E-03
```

Fig. 29. EAPROF Standard Output Listing (Sheet 12 of 13)

```
*************************************LASER SCATTERING FUNCTIONS*****************************************

TRANSVERSE INDEX = 11

    INDEX  ANGLE(DEG)        F

      1       0.000      0.
      2      10.000      0.
      3      20.000      0.
      4      30.000      0.
      5      45.000      0.
      6      90.000      0.
      7     135.000      0.
      8     180.000      0.
```

Fig. 29. EAPROF Standard Output Listing (Sheet 13 of 13)

```
TITLE     MSP INVERSION -- G
ZDATA
RUN
```

Fig. 30. PARIC1 Input Data Listing

```
***************************************** SUMMARY INPUT LISTING ****************************************

E/A TAPE TITLE       MSP MODEL ANALYSIS -- AL203/H20
INVERSION TITLE      MSP INVERSION -- G
NUMBER OF ZONES                 10
PLUME RADIUS(CM)       1.000E+01
PARTICLE SPECIES             AL203
```

Fig. 31. PARIC1 Standard Output Listing (Sheet 1 of 2)

```
********************************** PARTICLE INVERSION RESULTS **********************************
INDEX     R(CM)       G(CM-1)

  1      0.           5.900E-03
  2      1.000E+00    5.900E-03
  3      2.000E+00    5.900E-03
  4      3.000E+00    5.900E-03
  5      4.000E+00    5.900E-03
  6      5.000E+00    5.900E-03
  7      6.000E+00    5.900E-03
  8      7.000E+00    5.900E-03
  9      8.000E+00    5.900E-03
 10      9.000E+00    5.900E-03
 11      1.000E+01    5.900E-03
```

Fig. 31. PARIC1 Standard Output Listing (Sheet 2 of 2)

```
TITLE       MSP INVERSION -- A,B,AND P
CONVERGE            30          0.1
NDATA               10
ZDATA
RDATA                            2
RUN
```

Fig. 32. PARIC2 Input Data Listing

```
************************************** SUMMARY INPUT LISTING **************************************

E/A TAPE TITLE        MSP MODEL ANALYSIS -- AL2O3/H2O
INVERSION TITLE              MSP INVERSION -- A,B,AND P
PLUME RADIUS(CM)              1.000E+01
NUMBER OF ZONES                      10
PARTICLE SPECIES                  AL2O3
NUMBER OF SLOS INTERVALS             10
MAX NUMBER OF ITERATIONS             30
PERCENT CONVERGENCE           1.000E-01
NUMBER OF LASER LOS                  11
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 1 of 13)

```
*********************************** PARTICLE INVERSION RESULTS ************************************
INDEX     R(CM)        ABS(CM-1)    SCAT(CM-1)   EXT(CM-1)

  1      0.            3.199E-04    5.580E-03    5.900E-03
  2      1.000E+00     3.200E-04    5.580E-03    5.900E-03
  3      2.000E+00     3.201E-04    5.580E-03    5.900E-03
  4      3.000E+00     3.201E-04    5.580E-03    5.900E-03
  5      4.000E+00     3.201E-04    5.580E-03    5.900E-03
  6      5.000E+00     3.201E-04    5.580E-03    5.900E-03
  7      6.000E+00     3.201E-04    5.580E-03    5.900E-03
  8      7.000E+00     3.201E-04    5.580E-03    5.900E-03
  9      8.000E+00     3.201E-04    5.580E-03    5.900E-03
 10      9.000E+00     3.201E-04    5.580E-03    5.900E-03
 11      1.000E+01     3.200E-04    5.580E-03    5.900E-03
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 2 of 13)

RINDEX= 1 RADIUS(CM)= 0.

INDEX	ANG(DEG)	P(SR-1)
1	0.00	6.084E+00
2	10.00	5.468E+00
3	20.00	4.096E+00
4	30.00	2.764E+00
5	45.00	1.478E+00
6	90.00	4.275E-01
7	135.00	2.975E-01
8	180.00	9.583E-01

Fig. 33. PARIC2 Standard Output Listing (Sheet 3 of 13)

RINDEX= 2 RADIUS(CM)= 1.000E+00

INDEX	ANG(DEG)	P(SR-1)
1	0.00	6.084E+00
2	10.00	5.468E+00
3	20.00	4.096E+00
4	30.00	2.764E+00
5	45.00	1.478E+00
6	90.00	4.275E-01
7	135.00	2.975E-01
8	180.00	9.584E-01

Fig. 33. PARIC2 Standard Output Listing (Sheet 4 of 13)

RINDEX= 3 RADIUS(CM)= 2.000E+00

INDEX	ANG(DEG)	P(SR-1)
1	0.00	6.084E+00
2	10.00	5.468E+00
3	20.00	4.096E+00
4	30.00	2.764E+00
5	45.00	1.478E+00
6	90.00	4.275E-01
7	135.00	2.975E-01
8	180.00	9.584E-01

Fig. 33. PARIC2 Standard Output Listing (Sheet 5 of 13)

RINDEX= 4 RADIUS(CM)= 3.000E+00

INDEX	ANG(DEG)	P(SR-1)
1	0.00	6.084E+00
2	10.00	5.[illegible]68E+00
3	20.00	4.0[illegible]6E+00
4	30.00	2.764E+00
5	45.00	1.478E+00
6	90.00	4.275E-01
7	135.00	2.975E-01
8	180.00	9.584E-01

Fig. 33. PARIC2 Standard Output Listing (Sheet 6 of 13)

```
RINDEX=  5     RADIUS(CM)= 4.000E+00

INDEX    ANG(DEG)      P(SR-1)

  1         0.00      6.084E+00
  2        10.00      5.468E+00
  3        20.00      4.096E+00
  4        30.00      2.764E+00
  5        45.00      1.478E+00
  6        90.00      4.275E-01
  7       135.00      2.975E-01
  8       180.00      9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 7 of 13)

RINDEX= 6 RADIUS(CM)= 5.000E+00

INDEX	ANG(DEG)	P(SR-1)
1	0.00	6.084E+00
2	10.00	5.468E+00
3	20.00	4.096E+00
4	30.00	2.764E+00
5	45.00	1.478E+00
6	90.00	4.275E-01
7	135.00	2.975E-01
8	180.00	9.584E-01

Fig. 33. PARIC2 Standard Output Listing (Sheet 8 of 13)

```
RINDEX=  7    RADIUS(CM)= 6.000E+00

INDEX   ANG(DEG)      P(SR-1)

  1        0.00      6.084E+00
  2       10.00      5.468E+00
  3       20.00      4.096E+00
  4       30.00      2.764E+00
  5       45.00      1.478E+00
  6       90.00      4.275E-01
  7      135.00      2.975E-01
  8      180.00      9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 9 of 13)

```
RINDEX=  8    RADIUS(CM)= 7.000E+00

INDEX    ANG(DEG)       P(SR-1)

  1         0.00       6.084E+00
  2        10.00       5.468E+00
  3        20.00       4.096E+00
  4        30.00       2.764E+00
  5        45.00       1.478E+00
  6        90.00       4.275E-01
  7       135.00       2.975E-01
  8       180.00       9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 10 of 13)

```
RINDEX=  9     RADIUS(CM)= 8.000E+00

INDEX    ANG(DEG)       P(SR-1)

  1        0.00       6.084E+00
  2       10.00       5.468E+00
  3       20.00       4.096E+00
  4       30.00       2.764E+00
  5       45.00       1.478E+00
  6       90.00       4.275E-01
  7      135.00       2.975E-01
  8      180.00       9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 11 of 13)

```
RINDEX= 10     RADIUS(CM)= 9.000E+00

INDEX    ANG(DEG)       P(SR-1)

  1         0.00       6.084E+00
  2        10.00       5.468E+00
  3        20.00       4.096E+00
  4        30.00       2.764E+00
  5        45.00       1.478E+00
  6        90.00       4.275E-01
  7       135.00       2.975E-01
  8       180.00       9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 12 of 13)

```
RINDEX= 11     RADIUS(CM)= 1.000E+01

INDEX    ANG(DEG)       P(SR-1)

  1          0.00       6.084E+00
  2         10.00       5.468E+00
  3         20.00       4.096E+00
  4         30.00       2.764E+00
  5         45.00       1.478E+00
  6         90.00       4.275E-01
  7        135.00       2.975E-01
  8        180.00       9.584E-01
```

Fig. 33. PARIC2 Standard Output Listing (Sheet 13 of 13)

```
TITLE      MSP INVERSION -- TP
CONVERGE          30          0.1
PLMDATA           3.          57.         800.        0.75
NDATA             16          10
ZDATA
RDATA                         2           2           2
RUN
```

Fig. 34. PARIC3 Input Data Listing

```
************************************* SUMMARY INPUT LISTING *************************************

E/A TAPE TITLE                      MSP MODEL ANALYSIS -- AL203/H20
INVERSION TITLE                     MSP INVERSION -- TP
PLUME RADIUS(CM)                     1.000E+01
NUMBER OF ZONES                             10
PARTICLE SPECIES                         AL203
NUMBER OF SLOS INTERVALS                    10
MAX NUMBER OF ITERATIONS                    30
PERCENT CONVERGENCE                  1.000E-01
DISTANCE TO NOZZLE PLANE(CM)         3.000E+00
DISTANCE TO END PLANE(CM)            5.700E+01
NOZZLE TEMPERATURE(DEGK)             8.000E+02
NOZZLE EMISSIVITY                    7.500E-01
ANGLE INTEGRATION GRID ID
NUMBER OF SCATTERING ANGLES                 11
NUMBER OF AZIMUTHAL ANGLES                  16
WAVENUMBER(CM-1)                     3.985E+03
INTAPE FOR ABS COEFFICIENT                   2
INTAPE FOR SCAT COEFFICIENT                  2
INTAPE FOR PHASE FUNCTION                    2
```

Fig. 35. PARIC3 Standard Output Listing (Sheet 1 of 2)

```
********************************** PARTICLE INVERSION RESULTS **********************************
INDEX     R(CM)        TP(DEGK)

   1     0.            1.500E+03
   2     1.000E+00     1.475E+03
   3     2.000E+00     1.435E+03
   4     3.000E+00     1.365E+03
   5     4.000E+00     1.280E+03
   6     5.000E+00     1.165E+03
   7     6.000E+00     1.035E+03
   8     7.000E+00     8.801E+02
   9     8.000E+00     7.641E+02
  10     9.000E+00     7.539E+02
  11     1.000E+01     7.415E+02
```

Fig. 35. PARIC3 Standard Output Listing (Sheet 2 of 2)

```
CONVERGE            33          0.1         0.1
PLMDATA             3.          57.         800.        0.75
NDATA               16          10
GPARAM
   NERDH 20W     3985.0         300.        0.87394     6.53        1.00        10.
       100.   .4262E+00   .1242E+00
       200.   .2606E+00   .2722E+00
       300.   .1972E+00   .4058E+00
       400.   .1614E+00   .5306E+00
       500.   .1256E+00   .6658E+00
       600.   .1077E+00   .7770E+00
       700.   .8985E-01   .8926E+00
       800.   .7672E-01   .9990E+00
       900.   .6836E-01   .1096E+01
      1000.   .6000E-01   .1189E+01
      1100.   .5460E-01   .1310E+01
      1200.   .4920E-01   .1452E+01
      1300.   .4380E-01   .1592E+01
      1400.   .3839E-01   .1738E+01
      1500.   .3299E-01   .1893E+01
      1600.   .3057E-01   .2359E+01
      1700.   .2815E-01   .2828E+01
      1800.   .2573E-01   .3313E+01
      1900.   .2331E-01   .3822E+01
      2000.   .2089E-01   .4363E+01
      2100.   .1980E-01   .5327E+01
      2200.   .1871E-01   .6312E+01
      2300.   .1762E-01   .7323E+01
      2400.   .1652E-01   .8362E+01
      2500.   .1543E-01   .9435E+01
      2600.   .1499E-01   .1057E+02
      2700.   .1455E-01   .1169E+02
      2800.   .1411E-01   .1283E+02
      2900.   .1367E-01   .1397E+02
      3000.   .1323E-01   .1511E+02
      3100.   .1284E-01   .1665E+02
      3200.   .1246E-01   .1836E+02
      3300.   .1209E-01   .2027E+02
      3400.   .1174E-01   .2241E+02
      3500.   .1140E-01   .2479E+02
      3600.   .1107E-01   .2745E+02
      3700.   .1075E-01   .3043E+02
      3800.   .1044E-01   .3378E+02
      3900.   .1015E-01   .3753E+02
      4000.   .9861E-02   .4175E+02
ZDATA
RDATA                          2           2           2           2
TITLE       MSP INVERSION -- TG AND CG -- GAS
CALCDATA          GAS     LORENTZ          CG
RUN
TITLE       MSP INVERSION -- TG AND CG -- FOOB
CALCDATA          FOOB    LORENTZ          CG
RUN
TITLE       MSP INVERSION -- TG AND CG -- BOTH
CALCDATA          BOTH    LORENTZ          CG
RUN
```

Fig. 36. GASIC Input Data Listing

```
************************************** SUMMARY INPUT LISTING **************************************

E/A TAPE TITLE                      MSR MODEL ANALYSIS -- AL203/H20
INVERSION TITLE                     MSP INVERSION -- TG AND CG -- GAS
PLUME RADIUS(CM)                     1.000E+01
NUMBER OF ZONES                             10
GAS SPECIES                                H20
PARTICLE SPECIES                         AL203
GAS WAVENUMBER(CM-1)                 3.985E+03
PARTICLE WAVENUMBER(CM-1)            3.985E+03
GAS/PARTICLE MODE                          GAS
LINESHAPE                              LORENTZ
INHOMOGENEITY APPROXIMATION                 CG
BAND MODEL PARAMETERS IDNAME          NERDH20W
MAX NUMBER OF ITERATIONS                    30
PERCENT CONVERGENCE(TG)              1.000E-01
PERCENT CONVERGENCE(CG)              1.000E-01
DISTANCE TO NOZZLE PLANE(CM)         3.000E+00
DISTANCE TO END PLANE(CM)            5.700E+01
NOZZLE TEMPERATURE(DEGK)             8.000E+02
NOZZLE EMISSIVITY                    7.500E-01
NUMBER OF SCATTERING ANGLES                 11
NUMBER OF AZIMUTHAL ANGLES                  16
NUMBER OF SLOS INTERVALS                    10
INTAPE FOR ABS COEFFICIENT                   2
INTAPE FOR SCAT COEFFICIENT                  2
INTAPE FOR PHASE FUNCTION                    2
INTAPE FOR PARTICLE TEMP                     2
```

Fig. 37. GASIC Standard Output Listing (Sheet 1 of 6)

************************************* GAS INVERSION RESULTS *************************************

INDEX	R(CM)	TG(DEGK)	CG(MF)
1	0.	1.232E+03	3.355E-01
2	1.000E+00	1.220E+03	3.358E-01
3	2.000E+00	1.199E+03	3.354E-01
4	3.000E+00	1.160E+03	3.338E-01
5	4.000E+00	1.108E+03	3.303E-01
6	5.000E+00	1.032E+03	3.258E-01
7	6.000E+00	9.401E+02	3.057E-01
8	7.000E+00	8.254E+02	2.905E-01
9	8.000E+00	7.371E+02	2.732E-01
10	9.000E+00	7.273E+02	2.561E-01
11	1.000E+01	7.140E+02	2.365E-01

Fig. 37. GASIC Standard Output Listing (Sheet 2 of 6)

```
********************************** SUMMARY INPUT LISTING **********************************

E/A TAPE TITLE                       MSP MODEL ANALYSIS -- AL203/H20
INVERSION TITLE                      MSP INVERSION -- TG AND CG -- FOOB
PLUME RADIUS(CM)                      1.000E+01
NUMBER OF ZONES                              10
GAS SPECIES                                 H20
PARTICLE SPECIES                          AL203
GAS WAVENUMBER(CM-1)                  3.985E+03
PARTICLE WAVENUMBER(CM-1)             3.985E+03
GAS/PARTICLE MODE                          FOOB
LINESHAPE                               LORENTZ
INHOMOGENEITY APPROXIMATION                  CG
BAND MODEL PARAMETERS IDNAME           NERDH20W
MAX NUMBER OF ITERATIONS                     30
PERCENT CONVERGENCE(TG)               1.000E-01
PERCENT CONVERGENCE(CG)               1.000E-01
DISTANCE TO NOZZLE PLANE(CM)          3.000E+00
DISTANCE TO END PLANE(CM)             5.700E+01
NOZZLE TEMPERATURE(DEGK)              8.000E+02
NOZZLE EMISSIVITY                     7.500E-01
NUMBER OF SCATTERING ANGLES                  11
NUMBER OF AZIMUTHAL ANGLES                   16
NUMBER OF SLOS INTERVALS                     10
INTAPE FOR ABS COEFFICIENT                    2
INTAPE FOR SCAT COEFFICIENT                   2
INTAPE FOR PHASE FUNCTION                     2
INTAPE FOR PARTICLE TEMP                      2
```

Fig. 37. GASIC Standard Output Listing (Sheet 3 of 6)

```
*********************************** GAS INVERSION RESULTS ***********************************
INDEX     R(CM)        TG(DEGK)      CG(MF)

  1       0.           1.487E+03     1.475E-01
  2       1.000E+00    1.463E+03     1.477E-01
  3       2.000E+00    1.424E+03     1.480E-01
  4       3.000E+00    1.356E+03     1.484E-01
  5       4.000E+00    1.273E+03     1.488E-01
  6       5.000E+00    1.161E+03     1.492E-01
  7       6.000E+00    1.033E+03     1.496E-01
  8       7.000E+00    8.827E+02     1.499E-01
  9       8.000E+00    7.693E+02     1.508E-01
 10       9.000E+00    7.585E+02     1.509E-01
 11       1.000E+01    7.457E+02     1.507E-01
```

Fig. 37. GASIC Standard Output Listing (Sheet 4 of 6)

```
************************************** SUMMARY INPUT LISTING ***************************************

E/A TAPE TITLE                     MSP MODEL ANALYSIS -- AL203/H20
INVERSION TITLE                    MSP INVERSION -- TG AND CG -- BOTH
PLUME RADIUS(CM)                    1.000E+01
NUMBER OF ZONES                            10
GAS SPECIES                               H20
PARTICLE SPECIES                        AL203
GAS WAVENUMBER(CM-1)                3.985E+03
PARTICLE WAVENUMBER(CM-1)           3.985E+03
GAS/PARTICLE MODE                        BOTH
LINESHAPE                             LORENTZ
INHOMOGENEITY APPROXIMATION                CG
BAND MODEL PARAMETERS IDNAME         NERDH20W
MAX NUMBER OF ITERATIONS                   30
PERCENT CONVERGENCE(TG)             1.000E-01
PERCENT CONVERGENCE(CG)             1.000E-01
DISTANCE TO NOZZLE PLANE(CM)        3.000E+00
DISTANCE TO END PLANE(CM)           5.700E+01
NOZZLE TEMPERATURE(DEGK)            8.000E+02
NOZZLE EMISSIVITY                   7.500E-01
NUMBER OF SCATTERING ANGLES                11
NUMBER OF AZIMUTHAL ANGLES                 16
NUMBER OF SLOS INTERVALS                   10
INTAPE FOR ABS COEFFICIENT                  2
INTAPE FOR SCAT COEFFICIENT                 2
INTAPE FOR PHASE FUNCTION                   2
INTAPE FOR PARTICLE TEMP                    2
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Fig. 37. GASIC Standard Output Listing (Sheet 5 of 6)

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************************************** GAS INVERSION RESULTS **************************************
INDEX     R(CM)        TG(DEGK)      CG(MF)

  1     0.             1.499E+03    1.499E-01
  2     1.000E+00      1.474E+03    1.499E-01
  3     2.000E+00      1.434E+03    1.499E-01
  4     3.000E+00      1.364E+03    1.498E-01
  5     4.000E+00      1.280E+03    1.498E-01
  6     5.000E+00      1.165E+03    1.497E-01
  7     6.000E+00      1.035E+03    1.497E-01
  8     7.000E+00      8.803E+02    1.496E-01
  9     8.000E+00      7.641E+02    1.498E-01
 10     9.000E+00      7.541E+02    1.500E-01
 11     1.000E+01      7.424E+02    1.500E-01
```

Fig. 37. GASIC Standard Output Listing (Sheet 6 of 6)

APPENDIX

REVISED ABEL INVERSION PROCEDURE

A transverse function f(z) defined in terms of a radial function g(r) by

$$f(z) = 2 \int_z^R g(r) \frac{rdr}{\left(r^2 - z^2\right)^{1/2}} \quad (A1)$$

can be inverted to obtain g(r) with the Abel inversion

$$g(r) = -\frac{1}{2\pi r} \frac{dF(r)}{dr} \quad (A2)$$

where

$$F(r) = 2 \int_r^R f(z) \frac{xdx}{\left(z^2 - r^2\right)^{1/2}} . \quad (A3)$$

In the previously developed gas-only inversion code EMABIC (Ref. 3), evaluation of the integral form

$$F(y) = 2 \int_y^R G(x) \frac{xdx}{\left(x^2 - y^2\right)^{1/2}} \quad (A4)$$

used in both Eqs. (A1) and (A3) was performed with a quadrature formula of the form

$$F_k = \Delta \left[\alpha_1(k) G_k + \sum_{n=k+1}^{N} \alpha_2(k,n) G_n + \alpha_3(k) G_{N+1}\right] \quad (A5)$$

(with the summation deleted for $k = N$ and $F_k \equiv 0$ for $k = N + 1$). In Eq. (A5), the α's are quadrature coefficients and F_k and G_n are function values at the quadrature grid points. R is the cylindrical source radius, N is the number of equal-sized slabs assumed in the quadrature approximation, and $\Delta = R/N$. The α coefficients in Eq. A5 were derived with the assumption that G(x) in Eq. (A4) varied as

$$G(x)\ A + Bx^2 \tag{A6}$$

between grid points.

The use of Eq. (A5) to evaluate the integrals in both Eqs. (A1) and (A3) is somewhat inconsistent. For example, if the quadratic form of Eq. (A6) is used to approximate g(r) in the last radial zone (i.e., between points N and N + 1), then Eq. (A1) predicts that f(z) should have the form

$$f(z) = 2(A + Bz^2)\ \left(R^2 - z^2\right)^{1/2} + \frac{2B}{3}\,(R^2 - z^2)^{3/2} \tag{A7}$$

in the last transverse zone. In using Eq. (A3) for inversion, however, f(z) is implicitly assumed to vary in the last zone as

$$f(z) = a + bz^2, \tag{A8}$$

[i.e., since Eq. (A5) is used]. Although a and b can be selected so that Eq. (A8) fits Eq. (A7) reasonably well, it is not good enough to give an accurate recovery for g(r). This lack of ability to make a good fit occurs mainly in the outer zone and is caused by the fact that the correct form, Eq. (A7), for f(z) is very sharply bent in the region around z = R. The derivative f'(z) given by Eq. (A7) goes to $-\infty$ at $z = R$ while the implicitly used form Eq. (A8) could never achieve this slope. For inner zones, the form Eq. (A8) can be very much more accurately chosen to fit the true form of $f(z)$.

The accuracy of recovery in the last zone is further reduced because a backward difference approximation using only the points for $F(r)$ at N and $N + 1$ must be used to evaluate $g(r)$ from Eq. (A2). For inner zones, a more accurate three-point, central-difference formula can be used.

All told, the formulation as presented so far is adequate for profile generation and for inversion except in the outermost (and sometimes the second-to-last) zone.

This problem did not show up in the gas-only code EMABIC because a straightforward Abel inversion of transverse profiles was never made. Rather, Abel inversion was made on the difference of two transverse profiles, which, in the limit of convergence, approached zero. In the particle-only inversion code PARIC1, however, the principal output $\gamma(r)$ is obtained as a direct Abel inversion of $-\ell n\ \tau_p(z)$, and serious retrieval errors were obtained.

The first approach to solving this problem was to retain the quadratic form of Eq. (A6) for $g(r)$ in evaluating $f(z)$ with Eq. (A1) [i.e., in evaluating F_k with Eq. (A5)] but to derive new α-coefficients based on the variation of Eq. (A7) for evaluating $F(r)$ in Eq. (A3). This approach was soon abandoned because the resulting integrals were too complicated.

The final approach was a straightforward back-solution of Eq. (A5). Assume that G_n $(n = k + 1,\dots, N + 1)$ have been found. Then, from Eq. (A5), G_k $(k = 1,\dots,N)$ can be obtained by

$$G_k = \frac{1}{\alpha_1(k)} \left[\frac{F_k}{\Delta} - \sum_{n=k+1}^{N} \alpha_2(k,n)\ G_n - \alpha_3(k)\ G_{N+1}\right] \qquad \text{(A9)}$$

(Note: the summation term is deleted for $k = N$). The solutions for G_k at $k = N + 1$ is determined by fitting the known true form Eq. (A7) to the points F_{N-1} and F_N by

$$F_{N-1} = 2\left(A + Bz^2_{N-1}\right)\left(R^2 - z^2_{N-1}\right)^{1/2} + \frac{2B}{3}\left(R^2 - z^2_{N-1}\right)^{3/2}$$

$$F_N = 2\left(A + Bz_N^2\right)\left(R^2 - Bz_N^2\right)^{1/2} + \frac{2B}{3}\left(R^2 - z_N^2\right)^{3/2}. \qquad \text{(A10)}$$

With $z_{N-1} = R - 2\Delta$ and $z_N = R - \Delta$, these two equations can be solved for A and B. When determined, they are substituted into Eq. (A6) and G_{N+1} determined by evaluation at $x = R$. The result is

$$G_{N+1} = \frac{8(N-1)^{3/2}\, F_N - (2N-1)^{3/2}\, F_{N-1}}{4\Delta(2N-1)\ \sqrt{(2N-1)\,(N-1)}}$$

Equations (A9) and (A10) then defined an inversion procedure for obtaining the radial function G(r) from the transverse function F(z).